7 Heat Capacity and Heat Transfer

A. Heat capacity
B. Specific heat
C.. Heat capacities and thermal interaction
D. Heat capacity and entropy
E. Isothermal and adiabatic processes of ideal gases
F. Heat transfer
G. Summary

Large-scale systems can usefully be characterized by some thermal properties relating heat and temperature. This chapter discusses one such important property, "heat capacity", that characterizes any system or substance in equilibrium. It will also briefly mention another property, "thermal conductivity", that specifies the rate at which heat is transferred when a temperature difference exists in a non-equilibrium situation.

A. Heat capacity

Definition of heat capacity. When a small amount of heat d'Q is absorbed by a system, its absolute temperature changes by a corresponding small amount dT. The relation between d'Q and dT is conveniently described by a quantity called the system's *heat capacity*.

Def:

> **Heat capacity:** The heat capacity C_y of a system (when some other property y of the system remains constant) is a quantity describing the relation between the heat d'Q absorbed by the system and its corresponding temperature change dT. Quantitatively it is defined as the ratio
>
> $$C_y = \left(\frac{d'Q}{dT}\right)_y.$$

(A-1)

The symbol d' (with a prime ornamenting the letter d) is here used to denote *any* infinitesimal quantity even if it is not an infinitesimal difference. Thus d'Q is just an infinitesimally small amount of heat, but does not denote any difference.

The quantity y which is kept fixed has here been explicitly indicated by the subscript. (For example, this quantity might be the system's volume V or its pressure p.)

The definition (A-1) implies that

$$d'Q = C\, dT \qquad\qquad (A\text{-}2)$$

(where we have not bothered to indicate explicitly what other quantity y is kept constant).

For example, if a system's heat capacity C is large, a given small temperature change dT requires a larger amount of absorbed heat d'Q than if the

system's heat capacity is small. Conversely, (A-2) implies that dT = d'Q/C. If the heat capacity is large, a given small heat d'Q absorbed by the system produces thus a smaller temperature change dT than if the system's heat capacity is small.

Units of heat capacity. According to its definition (A-1), the units of heat capacity are those of a heat divided by an absolute temperature, i.e., the units are joule/kelvin.

Temperature dependence of the heat capacity. Ordinarily, the heat capacity C itself depends on the absolute temperature T and the other properties of the system. For instance, suppose that a larger amount of heat is required to raise a system's temperature from 500 K to 501 K than is required to raise its temperature from 300 K to 301 K. Then the heat capacity of the system would be larger at 500 K than at 300 K.

For example, Fig. A-1 shows how the heat capacity of a piece of copper metal varies with its absolute temperature. (Note that this heat capacity becomes nearly constant at temperatures near to, or above, room temperature.)

According to (A-2), the relation d'Q = C dT is valid at any temperature T. The heat capacity C at this temperature relates then any small temperature change dT to the correspondingly small heat d'Q absorbed by the system.

Suppose that one wants to find the total heat Q absorbed by the system when its temperature changes by a *large* amount, from some initial temperature T_A to some final temperature T_B. Then one needs only to add up the small amounts of heat absorbed by the system as its temperature is changed by successive small amounts. Thus the total heat absorbed is simply equal to the following sum

Fig. A-1. Temperature dependence of the heat capacity, at constant volume, of a mole of copper metal.

$$Q = \int_A^B d'Q = \int_A^B C \, dT \; . \qquad \text{(A-3)}$$

In the special case where the heat capacity is temperature-independent, C is merely a constant factor in this sum. Then (A-3) yields the following simple result:

$$\textit{If C is constant,} \qquad Q = C \int_A^B dT = C \, (T_B - T_A) \qquad \text{(A-4)}$$

since the sum of the successive small temperature changes is just equal to the total temperature change $(T_B - T_A)$.

Heat capacity at constant volume. Suppose that the volume V of a system is kept constant. (For example, suppose that the system is a gas within a container of fixed size.) Then the heat capacity C_V at constant volume is defined as the ratio

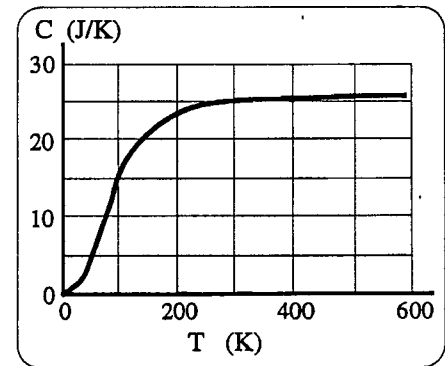$$C_V = \left(\frac{d'Q}{dT}\right)_V . \qquad \text{(A-5)}$$

relating the heat d'Q absorbed under these conditions to the resulting temperature change dT. But, no work is done on the system if its volume is kept fixed (and no other work is done on it by rotating paddle wheels or other means). Hence the thermodynamic energy law implies that

$$dE = d'W + d'Q = 0 + d'Q, \qquad (A\text{-}6)$$

so that the heat d'Q absorbed by the system is just equal to the change dE of its internal energy. Hence (A-5) is simply equal to

$$C_V = \left(\frac{dE}{dT}\right)_V. \qquad (A\text{-}7)$$

The heat capacity at constant volume describes, therefore, the relation between the energy change dE and temperature change dT of a system. Correspondingly, (A-7) implies the relation

To indicate explicitly that the volume is kept constant when the temperature is changed, (A-7) could also be written as the *partial derivative* $(\partial E/\partial T)_V$. But this is not necessary since the subscript V already indicates that the volume is to be kept fixed.

$$dE = C_V \, dT. \qquad (A\text{-}8)$$

As pointed out in Chapter 4, the absolute temperature of a system increases when its internal energy increases. Accordingly, (A-7) implies that the heat capacity at constant volume of any such system must be *positive*.

Heat capacity at constant pressure. In many cases, it is the pressure (rather than the volume) which remains constant when heat is absorbed by a system. For example, when heat is absorbed by a cup of tea in a room, or to a gas within a balloon in the room, the pressure of the system remains equal to the surrounding atmospheric pressure.

When a small amount of heat d'Q is absorbed under conditions of constant pressure p, some work d'W = –p dV is done on the system if the volume of the system changes by some amount dV. The thermodynamic energy law then implies that

$$dE = d'W + d'Q$$

or
$$d'Q = dE - d'W = dE + p \, dV. \qquad (A\text{-}9)$$

Suppose that the temperature of a system changes by a small amount dT. The heat d'Q absorbed by the system when its pressure is kept constant is thus different than when its volume is kept constant (because work is done by the pressure in the first case, but not in the second). Correspondingly, the heat capacity C_p at constant pressure

$$C_p = \left(\frac{dE}{dT}\right)_p \qquad (A\text{-}10)$$

is *different* from the heat capacity C_V at constant volume specified by (A-7).

In the case of a solid or liquid, the volume change resulting from a small temperature change is very small so that the work done by the surrounding pressure is nearly negligible. Hence the heat capacity C_p at constant pressure is nearly the same as the heat capacity C_V at constant volume. But in the case of a

gas, such a small temperature change results in an appreciable change of volume. Correspondingly, the heat capacity C_p at constant pressure of a gas is appreciably different from its heat capacity C_V at constant volume.

Problems

[A-1] *Heat capacity of a lead block*

It is found that 195 J of heat must be absorbed by a 3.00 kg lead block to raise its temperature from 22.00 °C to 22.50 °C.

(a) By what amount does the absolute temperature of the lead block change in this process?
(b) What is the heat capacity of this lead block?
(c) What would be the heat capacity of a lead block having a mass of 1.00 kg?
    <h-5> <a-8>

[A-2] *Heating a cup of water*

To make tea, 0.20 liters of water in a cup needs to be heated from a temperature of 25 °C to 100 °C. The heat capacity of this amount of water is approximately constant in this temperature range and equal to 840 J/K.

(a) How much heat must be absorbed by the water to change its temperature by the desired amount?
(b) An electrical heating coil supplies heat to the water at the rate of 500 watt (i.e., 500 joule/second). How long a time is required for this coil to heat the water by the desired amount? *<a-3>*

[A-3] *Comparing temperature changes and absorbed heats of two systems*

The heat capacity C_X of a system X is 100 times larger than the heat capacity C_Y of a system Y.

(a) Suppose that the same amount of heat is added to both systems. How much larger then is the temperature rise of system X compared to the temperature rise of system Y?
(b) Suppose that one wishes to add heat to each system so as to raise the temperature of each by the same amount. How much larger then must be the heat added to system X compared to the heat added to system Y?
    <a-10>

[A-4] *Heat capacities of a copper block*

The heat capacity of a copper block at room temperature (293 K) is 350 J/K. The heat capacity of the same block at the temperature of liquid nitrogen (77 K) is 170 J/K.

(a) How much heat must be absorbed by this block, originally at liquid-nitrogen temperature, to raise its temperature by 1.0 K?
(b) Suppose that this same amount of heat is absorbed by the block when it is originally at room temperature. When then is the resulting increase in the temperature of the block? *<a-6>*

[A-5] *Comparison of C_V and C_p for an ideal gas*

The heat capacity of an ideal gas, maintained at constant volume, is C_V. Suppose that the absolute temperature of this gas is increased by a small amount dT.

(a) What is the resulting heat absorbed by the gas? What is the resulting increase of the internal energy of the gas?

Suppose that this gas is, instead, maintained at the constant pressure of the surrounding atmosphere so that its volume can change. Under these conditions the absolute temperature of the gas is again increased by the same small amount dT as before.

(b) Is the resulting increase of the internal energy of the gas then larger than, smaller than, or the same as under the previous conditions where the volume of the gas remained constant? Why? <h-2>

(c) Does the volume of the gas then increase, decrease, or remain the same? Why?

(d) Is the work done on the gas by the surrounding atmosphere then positive, negative, or zero?

(e) Is the heat absorbed by the gas under these conditions of constant pressure larger than, smaller than , or the equal to the heat absorbed by the gas under the previous conditions of constant volume? Why? <h-7>

(f) Is the heat capacity Cp of the gas at constant pressure larger than, smaller than, or equal to the heat capacity C_V of the gas at constant volume? <a-2>

B. Specific heat

Heat capacities and specific heats

Quantities proportional to the amount of a substance. A substance (like the water inside a container or the metal inside a copper block) is *homogeneous* because any region within such a substance has the same properties as any other region of the same size. Suppose that the size of a system, consisting of such a homogeneous substance, is three times larger than that of another system consisting of this substance. Then the number N of particles in this system will also be three times larger and its internal energy E will also be three times larger. Similarly, the amount of heat that needs to be absorbed by the system to raise its temperature by a degree (i.e., the heat capacity C of the system) will also be three times larger. In short, properties like the internal energy E or the heat capacity C of such a system are simply proportional to the amount of substance.

Specific heat per mole. By dividing any such quantity by the amount of the substance, one obtains a quantity which is characteristic of the substance but *independent of its amount*. For example, if one divides the specific heat C of ν moles of a substance by the number ν of moles, one obtains a quantity c called the *specific heat per mole* (or *molar specific heat*) of the substance.

Def:
$$\boxed{\textit{Specific heat per mole:} \quad c = \frac{C}{\nu}\,.} \qquad \text{(B-1)}$$

Once the specific heat c of a substance has been determined by measurements or calculation, the heat capacity of *any* amount of the substance can immediately be found. Indeed, the heat capacity C of ν moles of this substance is then simply equal to

$$C = \nu\, c. \qquad \text{(B-2)}$$

Specific heat per unit mass. Similarly, one may divide the heat capacity C of a mass M of the substance by this mass. Thus one obtains the specific heat $\underline{c}$ per unit mass of this substance.

Def:
$$\boxed{\textit{Specific heat per unit mass:} \quad \underline{c} = \frac{C}{M}} \qquad \text{(B-3)}$$

We use $\underline{c}$ (an underlined c) to distinguish the specific heat per unit mass from the specific heat c per mole.

This specific heat per unit mass is again a quantity characteristic of the substance, but independent of its amount. Once this specific heat $\underline{c}$ is known, the heat capacity of any mass M of the substance is then simply equal to

$$C = M\,\underline{c}. \qquad (B\text{-}4)$$

Fig. B-1 indicates the measured specific heats of some common substances.

Molar specific heat of an ideal gas

According to (A-7), the molar specific heat c_V, at *constant volume*, of any substance specifies how rapidly the internal energy of a mole of this substance increases with its absolute temperature. The specific heat of any such substance can thus readily be found if one knows how the internal energy of this substance depends on its absolute temperature.

Monatomic ideal gas. One mole of any substance contains N_a molecules (where N_a is Avogadro's number). A particularly simple such substance is a monatomic ideal gas. According to (6A-5), the internal energy E of one mole of such a monatomic gas is merely equal to its kinetic energy K, i.e.,

$$E = K = \frac{3}{2} N_a kT. = \frac{3}{2} RT \qquad (B\text{-}5)$$

where $R = N_a k$ is the gas constant. The heat capacity c_V at constant volume of one mole of such a monatomic gas is then, by (A-7) simply equal to

$$c_V = \left(\frac{dE}{dT}\right)_V = \frac{3}{2} R. \qquad (B\text{-}6)$$

Our theory thus predicts that the molar specific heat c_V of *any* monatomic ideal gas (e.g., of any gas such as helium, neon, argon, or xenon) has the same temperature-independent value equal to

$$\boxed{\textit{for any monatomic gas,} \quad c_V = \frac{3}{2} R = 12.5 \text{ J/(K mole)}} \qquad (B\text{-}7)$$

where we have used the value (6D-8) of the gas constant R.

Polyatomic ideal gas. The internal energy E of a polyatomic gas consists not only of the kinetic energy K of center-of-mass motions of all the molecules, but also of internal molecular energy E_{mol} (due to kinetic and potential energy of the atoms moving relative to each other within each molecule.) As indicated in (6A-7), one can thus write

$$E = K + E_{mol}. \qquad (B\text{-}8)$$

According to (A-10), the heat capacity c_V of a mole of such a polyatomic gas is again determined by how rapidly the internal energy E of the gas increases with its temperature. But now this energy increases not only because the kinetic energy K increases with temperature, but also because the internal molecular

Substance	c_p (J/mol K)	$\underline{c}_p$ (J/kg K)
Aluminum	24.6	910
Copper	24.8	390
Iron	26.3	470
Lead	26.9	130
Silver	25.3	234
Mercury	27.7	138
Salt (NaCl)	51.4	879
Water	75.4	4190
Ice	36.5	2000
Glass		840

Fig. B-1. Specific heats (at constant pressure) of some common substances at room temperature.

energy E_{mol} increases with temperature. Hence the specific heat c_V of a polyatomic gas is *larger* than that of a monatomic gas.

Problems

[B-1] *Heat capacity of copper*

The atomic weight of copper is 63.5 and the molar specific heat of copper metal is 24.8 J/mol K.

(a) What is the mass of one mole of copper metal?
(b) How many moles of copper are contained in one kilogram of copper metal?
(c) What is the specific heat per kilogram of copper metal?
(d) What is the heat capacity of a piece of copper having a mass of 0.200 kg?
    <a-30>

[B-2] *Specific heat and change of internal energy*

Iron metal near room temperature has a specific heat of 470 J/kg K which is nearly temperature-independent. The volume of the metal also varies negligibly with its temperature.

(a) What is the heat absorbed by 0.65 kg piece of iron when its temperature is increased by 25 K?
(b) What is the change of the internal energy of this piece of iron when its temperature is increased by this amount? *<a-19>*

[B-3] *Specific heat of a diatomic gas*

A diatomic gas (like nitrogen) consists of molecules each of which consists of two atoms. At an absolute temperature T, the average kinetic energy of center-of-mass motion of each of these molecules is $\frac{3}{2}kT$ (i.e., the same as the average kinetic energy of any monatomic gas molecule). However, each diatomic molecule has also an average kinetic energy of kT due to the rotation of its two atoms around their common center of mass.

(a) What is the total internal energy of a mole of diatomic gas at the temperature T? (Express your answer in terms of the gas constant R.)
(b) What is the molar specific heat, at constant volume, of such a diatomic gas? (Express your answer in terms of the gas constant R.)
(c) What is the numerical value of this specific heat? *<a-13>*

C. Heat capacities and thermal interaction

A knowledge of heat capacities allows one to examine in greater detail the thermal interaction between two systems.

For example, consider two systems X and X' which have initially respective temperatures T_i and T_i'. These systems, while isolated from their surroundings, are now brought into thermal contact with each other and finally reach equilibrium at a common final temperature T_f. (See Fig. C-1.) Because the energy of the total system remains constant, the energy changes of the two systems during their thermal interaction must be related so that

$$\Delta E + \Delta E' = 0. \tag{C-1}$$

Consider the simple situation where the volumes of these systems remain essentially constant so that no work is done on them by any surrounding
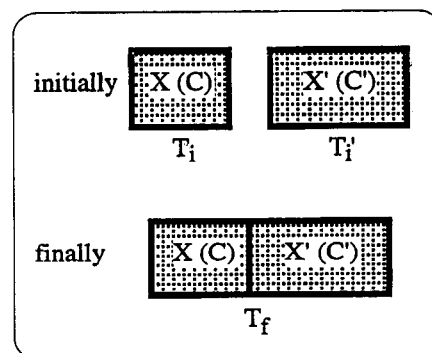

Fig. C-1. Thermal interaction between two systems.

pressure. (For example, these systems might be solids or liquids whose volumes change by negligible amounts when their temperatures change.) Suppose further that the heat capacities, at constant volume, of these systems are C and C' respectively, and essentially temperature independent. Then the energy change of each system is merely equal to the heat absorbed by it. Correspondingly, the energy relation (C-1) implies that

$$C\,(T_f - T_i) + C'\,(T_f - T_i') = 0. \tag{C-2}$$

This relation can be used to determine various quantities of interest, as indicated in the following paragraphs.

Determination of final temperature. Suppose that the heat capacities of the two systems are known. If the initial temperatures of the systems are known, the final temperature attained by the systems can then be predicted from (C-2) since this relation can be solved for the final temperature T_f. Thus

$$T_f = \frac{CT_i + C'T_i'}{C + C'}.$$

Comparative measurement of heat capacities. Alternatively, suppose that the heat capacity C of the system X is known. Then the heat capacity C' of any other system X' can be determined by measuring the initial temperatures of the systems and the final temperature attained by them after they have achieved mutual thermal equilibrium. Thus (C-2) implies that

$$C' = -C \left(\frac{T_f - T_i}{T_f - T_i'} \right).$$

Problems

[C-1] *Final temperature of hot water poured into a bucket*

A bucket, made of metal having a specific heat of 470 J/kg K, has a mass of 0.60 kg. Half a liter of hot water, having a mass of 0.50 kg, is initially at a temperature of 90°C and is poured into the bucket which is initially at a temperature of 20°C. The heat capacity of water is 4190 J/kg K. Negligible heat flows from the surroundings to the bucket and the water in it. What then is the final temperature of the water in the bucket?

The following illustrates the solution of this problem.

Sample solution

Situation: Bucket: M = 0.60 kg, $\underline{c}$ = 470 J/kg K, τ = 20°C.   **Analysis of problem**

                     Water: M = 0.50 Kg, $\underline{c}$ = 4190 J/kg K, τ = 90°C.

Goal: Final temperature τ' = ?

Apply energy law to system consisting of bucket and water: **Construction of solution**

$$(\Delta E)_{bucket} + (\Delta E)_{water} = 0. \tag{1}$$

Bucket:

   Initial state: τ = 20°C

⇓ $\Delta E = M\,\underline{c}\,\Delta T = (0.60\text{ kg}) (470\text{ J/kg K}) [(\tau' - 20°C)\,(K/°C)]$

   $\Delta E = 282\,(\tau' - 20°C)\,(J/°C) \tag{2}$

   Final state: Temperature τ'.

Neglect any small volume changes, i.e., any work done on the system by the surrounding atmosphere.

The arrow symbol ⇓ indicates the process leading between the states.

Water:

Initial state: $\tau = 90°C$

$\Downarrow$ $\Delta E = M \underline{c} \Delta T = (0.50 \text{ kg}) (4190 \text{ J/kg K}) [(\tau' - 90°C) (K/°C)]$

$\Delta E = 2095 (\tau' - 90°C) (\text{J/°C})$ (3)

Final state: Temperature τ'.

Putting (2) and (3) into (1):

$$282 (\tau' - 20°C) + 2095 (\tau' - 90°C) = 0$$

$$2377 \, \tau' = 194\,200 \,°C$$

$$\boxed{\tau' = 82°C}$$ (4)

[C-2] *Cooling a silver ornament in water*

A silver ornament, having a mass of 220 gram, has been immersed in a cleaning solution at 100°C. It is then cooled by being dropped into 120 grams of water, contained in an aluminum can having a mass of 150 gram. The water and can are initially at a temperature of 20°C and are thermally well insulated from their surroundings. What then is the final temperature of the ornament and the water? <a-21>

[C-3] *Measuring the specific heat of beryllium*

To measure the specific heat of beryllium (whose atomic weight is 9.01), an experimenter takes a 110 gram piece of the metal, at a temperature of 58.0°C, and drops it into a thermally insulated container filled with 250 grams of water. The initial temperature of the water and of the container (which has a heat capacity of 185 J/K) is 23.0°C. Its measured final temperature, after the beryllium has been dropped in, is 28.2°C.

(a) What is the specific heat per kilogram of beryllium metal near room temperature?
(b) What is the molar specific heat of beryllium metal near room temperature? <a-34>

[C-4] *Thermal interaction with a heat reservoir*

A system X is called a "heat reservoir" if its heat capacity C is very much larger than the heat capacity C' of any other system X' with which it may be placed into thermal contact. (For example, the water in a lake is a heat reservoir compared to any pebble that might be thrown into the lake.)

Suppose that such a heat reservoir, at an absolute temperature T_i, is actually brought into thermal contact with any such other system X' initially at an absolute temperature T_i'.

(a) What then is the final temperature of the heat reservoir after the systems have come to thermal equilibrium?
(a) What is the final temperature of the other system? <a-39>

[C-5] *Temperature measurement and thermometer heat capacity*

A thermally insulated aluminum cup, whose heat capacity is 55.0 J/K, is filled with 170 grams of ethyl alcohol (whose specific heat is 2430 J/kg·K). To measure the temperature of the alcohol, one immerses in it a thermometer having a heat capacity of 26.0 J/K. The thermometer is initially at the room temperature of 22.7°C. After it has come to equilibrium with the alcohol, the thermometer indicates a temperature of 52.4°C. What was the initial temperature of the alcohol? <a-23>

D. Heat capacity and entropy

A knowledge of a system's heat capacity also allows one to obtain information about its entropy.

Small entropy change. Consider any system of constant volume. The absolute temperature of a system specifies how rapidly the entropy S of the system increases with its energy E. Indeed, according to (4F-7) the absolute temperature T of the system is defined so that a small change dS of its entropy is given by

$$dS = \frac{dE}{T} = \frac{d'Q}{T}.$$
(D-1)

Here dE is the system's small energy change (or the small heat d'Q absorbed by the system) when its absolute temperature T is changed by a small amount dT. But d'Q = C dT if C is the system's heat capacity at constant volume. The small entropy change dS, resulting from a small temperature change dT, is thus

$$dS = \frac{C\,dT}{T}$$
(D-2)

where C is the system's heat capacity at the absolute temperature T.

Large entropy change. Suppose that one wants to find a system's change of entropy when its absolute temperature is changed from a temperature T_a to any other temperature T_b. Then one needs merely to change the system's temperature slowly enough (quasi-statically) so that it is at any instant essentially in equilibrium at some well-defined intermediate absolute temperature T. Any small entropy change of the system is then given by (D-2). Hence the system's total entropy change is just the sum of its small entropy changes during its successive small temperature changes. Hence

$$S_B - S_A = \int_{T_A}^{T_B} \frac{C\,dT}{T}.$$
(D-3)

The change of a system's entropy between any two temperatures can thus be found if one knows the system's heat capacity at all intermediate absolute temperatures.

Correspondingly, it is also possible to measure experimentally the change of a system's entropy. To do this, it is only necessary to measure its absolute temperature and its heat capacity at various temperatures — and then to use this information to calculate the sum (D-3).

Entropy change for constant heat capacity. The calculation of a system's entropy change becomes particularly simple if the system's heat capacity is independent of its temperature. In this case the heat capacity C in (D-3) is merely a constant factor which can be taken outside the sum. Thus (D-3) becomes

$$S_B - S_A = C \int_{T_A}^{T_B} \frac{dT}{T} = C \int_{T_A}^{T_B} d(\ln T)$$

since $dT/T = d(\ln T)$. But the sum of the successive small changes of the logarithm is just equal to the total change of this logarithm so that

$$S_B - S_A = C (\ln T_B - \ln T_A).$$

Thus,
> *if C is constant,* $\qquad S_B - S_A = C \ln\left(\frac{T_B}{T_A}\right).$

(D-4)

Problems

[D-1] *Entropy change of a heat reservoir*

A "heat reservoir" is a system whose heat capacity is so large that its temperature remains essentially constant irrespective of how much heat it absorbs from other systems. (For example, the air in a room is a heat reservoir compared to a cup of coffee introduced into the room.)

Consider a heat reservoir whose absolute temperature is T. What is the entropy change of this heat reservoir if it absorbs an amount of heat Q? *<a-18>*

[D-2] *Entropy change of a copper block*

Near room temperature, the molar specific heat of copper metal is nearly constant and equal to 24.8 J/mole K. The atomic weight of copper is 63.5.

(a) What is the heat capacity of a copper block which has a mass of 0.500 kg?

(b) When this block is dropped into boiling water, its temperature changes from room temperature (20°C) to 100°C. As a result, does the entropy of the copper block increase or decrease, and by what amount? *<a-5>*

[D-3] *Entropy change of melting lead*

Lead at atmospheric pressure melts at a temperature of 328°C. As the lead melts, its temperature remains constant and the volume of the metal remains nearly unchanged. But 2.47×10^5 J of heat must be absorbed by every kilogram of the metal to change it from its solid to its liquid form.

When one kilogram of lead melts, does its entropy increase or decrease? By what amount? *<a-17>*

[D-4] *Thermal interaction between identical systems*

Each of two identical systems has a temperature-independent heat capacity C. Initially, one of the systems has an absolute temperature T_1 and the other one has an absolute temperature T_2.

(a) What is the final temperature attained by these systems when they are brought into thermal contact and finally reach thermal equilibrium?

(b) What is the change of the total entropy of both systems between the initial situation and the final equilibrium situation? *<a-12>*

E. Isothermal and adiabatic processes of ideal gases

There are many cases where a gas is compressed or expanded by changing its volume. (For example, gasoline engines or steam engines work because gases contained in cylinders change their volumes and thus move pistons.) Quite often the change of volume occurs quasi-statically, i.e., sufficiently slowly that the gas is at any time in internal equilibrium (and thus characterized by a well-defined pressure and temperature uniform throughout the gas).

Since the molecules of a gas move with high speeds (several hundreds of meters per second), times of the order of milliseconds may actually be quite sufficient to maintain a gas in thermal equilibrium. A process occurring during times longer than a few milliseconds may thus be sufficiently slow to be considered quasi-static.

The process during which the volume of the gas is changed may, however, occur under various conditions. The following are some examples:

(a) The process may occur under *isothermal* conditions, i.e., under conditions where the temperature of the gas remains always the same. (This can be achieved by keeping the gas in thermal contact with another system which has a much larger heat capacity and which experiences thus negligible changes of temperature despite any heat exchanged with the gas.).

(b) The process may occur under *adiabatic* conditions, i.e., under conditions where the gas is thermally insulated from its surroundings. (This can be achieved by wrapping the gas container with fiberglass or some other insulating material.)

Let us now examine these processes in the simple case of an ideal gas.

Isothermal process

In this case the absolute temperature of the gas remains unchanged, i.e.,

$$T = \text{constant} \quad \text{or} \quad \Delta T = 0. \tag{E-1}$$

As the volume of the gas is changed, its internal energy remains unchanged (since the internal energy of an *ideal* gas depends only on its temperature, but not on its volume). The thermodynamic energy law then implies that

$$\Delta E = W + Q = 0 \tag{E-2}$$

so that
$$Q = -W. \tag{E-3}$$

For example, when the gas expands so that its volume increases, the work W done on the gas is negative. Hence positive heat is then absorbed by the gas from its surroundings in order to maintain the gas at its constant temperature.

The ideal-gas law implies that the pressure p of the gas is related to its volume V so that

$$p = \frac{\nu RT}{V}. \tag{E-4}$$

Since the temperature T of the gas remains constant in this isothermal process, the pressure of the gas is then inversely proportional to its volume. As the volume of the gas increases, its pressure thus decreases accordingly.

Adiabatic process

By contrast, consider the case where the gas is adiabatically isolated so that it cannot absorb or give off any heat. Thus

$$Q = 0 \qquad \text{(E-5)}$$

and the thermodynamic energy law implies that

$$\Delta E = W + Q = W. \qquad \text{(E-6)}$$

For example, when the gas expands so that its volume increases, the work W done on the gas is negative. Hence the internal energy E of the gas correspondingly decreases so that its absolute temperature T also decreases.

The pressure of the gas is still specified by the ideal-gas law (E-4). However, when the gas expands, its pressure now decreases for two reasons, both because its volume V increases and because its temperature T decreases. When the volume of the gas increases, its pressure thus decreases *more rapidly* than in the isothermal case where the temperature of the gas remains constant. (See Fig. E-1.)

Infinitesimal adiabatic process

We should like to determine quantitatively how the volume, pressure, and temperature of v moles of an ideal gas vary in an adiabatic process. To do this, it is simplest to consider first such a process in which the volume of the gas is slowly changed by an *infinitesimal* amount dV

Implications of the energy law. The thermodynamic energy law implies that

$$dE = d'W + d'Q = -p \, dV + 0 \qquad \text{(E-7)}$$

since the work done on the gas is $d'W = -p \, dV$ and since no heat is absorbed. If the gas is ideal, its small energy change dE does not depend on the change dV of its volume, but only on the change dT of its temperature. . Hence this energy change is the same as if the volume were kept constant. Thus it is simply related to the heat capacity of the gas *at constant volume*. Hence (E-7) implies that

$$v c_V dT = -p \, dV$$

or

$$dT = -\left(\frac{p}{v c_V}\right) dV \qquad \text{(E-8)}$$

where c_V is the molar specific heat of the gas at constant volume. This relation shows explicitly how the temperature of the gas varies as a result of a small change of its volume.

Implications of the ideal-gas law. We also know that the pressure, volume, and temperature of the gas are related by the ideal-gas law so that
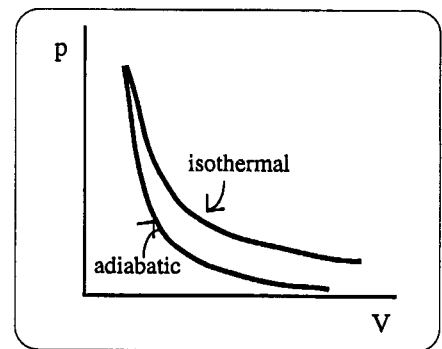
$$pV = v RT. \qquad \text{(E-9)}$$

Fig. E-1. Variation of gas pressure p with volume V under isothermal and adiabatic conditions.

When the volume of the gas changes by an infinitesimal amount, the quantities in (E-9) thus change correspondingly so that

$$d(pV) = \nu R\, dT$$

so that $\qquad\qquad V\, dp + p\, dV = \nu R\, dT.$ $\qquad\qquad$ (E-10)

The relations (E-8) and (E-10) now allow us to eliminate one of the three quantities V, p, or T in order to find a relation connecting the other two.

Adiabatic pressure-volume relation

Relation between small changes. For example, (E-8) relates the temperature change dT of the gas to its volume change dV. Substituting this result into (E-10), then yields

$$V\, dp + p\, dV = -R\left(\frac{p}{c_V}\right) dV$$

or $\qquad\qquad V\, dp + \gamma\, p\, dV = 0$ $\qquad\qquad$ (E-11)

where we have used γ (the Greek letter *gamma*) as a convenient abbreviation to denote the quantity

$$\boxed{\gamma = 1 + \frac{R}{c_V} = \frac{c_V + R}{c_V}\,.}$$ $\qquad$ (E-12)

If the heat capacity is temperature-independent, this quantity γ is merely a constant characterizing the particular gas. For example, in the case of a monatomic ideal gas, the molar specific heat [previously calculated in (B-7)] is $c_V = \frac{3}{2} R.$ Thus (E-12) implies that

$$\textit{for a monatomic ideal gas,} \qquad \gamma = \frac{5}{3} \approx 1.67.$$ $\qquad$ (E-13)

Relation between pressure and volume. The relation (E-11) between corresponding infinitesimal changes of pressure and volume can be readily used to relate these quantities themselves. Indeed, (E-11) can be divided by pV to yield

$$\frac{dp}{p} + \gamma\frac{dV}{V} = 0$$

so that the two terms involve p and V separately. By exploiting knowledge about logarithms, this can be written in the form

$$d(\ln p) + \gamma\, d(\ln V) = 0$$

$$d(\ln p) + d(\ln V^\gamma) = 0$$

$$d\,(\ln p + \ln V^\gamma) = 0$$

$$d\,[\ln (pV^\gamma)] = 0.$$

Hence the quantity ln (pV^γ) remains unchanged so that

> *for an adiabatic process,* pV^γ = constant.   (E-14)

This result specifies quantitatively how the pressure of an ideal gas varies with its volume in an adiabatic process.

   Comparison with an isothermal process.   B contrast, when the temperature of the gas is maintained constant, the ideal-gas law (E-9) implies that

   for an isothermal process, pV = constant.   (E-15)

By its definition (E-12), the constant γ is larger than 1. Hence (E-14) implies that the pressure of the gas varies more rapidly with its volume in an adiabatic process than in an isothermal process. This result is in accord with the qualitative conclusion reached earlier in this section and illustrated in Fig. E-1.

Problems

[E-1] *Pressure change due to compression of a gas*

   An ideal gas, which has a volume V_0 and a pressure p_0, is slowly compressed so that its final volume is $V_0/4$.

(a) Suppose that the gas is argon which is monatomic. What then is the final pressure of the gas if the compression is done isothermally? What is this final pressure if the compression is done adiabatically? (Express your answers in terms of the initial pressure p_0.)

(b) Answer the same questions if the gas is nitrogen which is diatomic and whose molar specific heat is $\frac{5}{2}$ R. *<a-38>*

[E-2] *Temperature change due to compression of a gas*

   Suppose that the gas in the preceding problem is initially at an absolute temperature T_0.

(a) If the gas is argon, what is its final absolute temperature after its compression if this is done isothermally? If this is done adiabatically? (Express your answers in terms of the initial absolute temperature T_0.)

(b) Answer the same questions is the gas is nitrogen. *<h-14> <a-25>*

[E-3] *Temperature-volume relation for an adiabatic ideal gas*

   An ideal gas is slowly compressed while it is thermally insulated from its surroundings. The absolute temperature T of the gas then varies with its volume V in such a fashion that TV^b = constant. What is the value of the constant b in this relation? Express your result in terms of the constant γ characterizing the gas. *<h-6> <a-42>*

F. Heat transfer and thermal conductivity

   Up to now we have been mostly concerned with equilibrium situations. By contrast, we now consider a simple non-equilibrium situation where heat is transferred from one system to another.

To be specific, Fig. F-1 illustrates two systems connected by a rod made of some metal or some other substance. If the absolute temperatures T and T' of these systems are the same, everything is in equilibrium and no energy will be transferred from one system to the other. But if the temperatures of these systems are different, energy in the form of heat will be transferred from one system to the other (until the systems ultimately reach equilibrium and attain a common temperature). What can one say about the rate of heat transfer $\hat{Q}$ (i.e., the amount of heat transferred per second) between the two systems when their temperatures are T and T'?

The rate of heat-transfer (or "heat flow") $\hat{Q}$ is zero if the temperature difference $\Delta T = T' - T = 0$. It is non-zero if $\Delta T \neq 0$ and is larger if this temperature difference is larger. The heat flow thus depends on the temperature difference in the manner qualitatively indicated in Fig. C-2. If the temperature difference is not too large, this graph is nearly straight so that one can write

$$\hat{Q} = (\text{constant}) \, \Delta T. \tag{F-1}$$

Here the constant depends on the nature of the connecting rod. If the cross-sectional area A of the rod is larger, one would expect the heat flow to be proportionately larger. But if the length L of the rod is larger, one would expect the heat flow to be proportionately smaller. Hence (F-1) can be written as

$$\hat{Q} = \kappa \frac{A}{L} \Delta T \tag{F-2}$$

where the constant κ (denoted by the Greek letter *kappa*) depends only on the nature of the substance of which the rod is made and is called the *thermal conductivity* of this substance. Fig. F-3 lists the measured thermal conductivities of a few common substances.

Problems

[F-1] *Heat flow through a glass window*

A window, 90 cm high and 50 cm wide, is made of 0.30 cm thick glass having a thermal conductivity of 0.80 W/m·K. Suppose that this window separates the inside of a room, at a temperature of 20°C, from the outside air having a temperature of –10°C. How much heat per second flows through this window from the room to the outside surroundings?

The following illustrates the solution of this problem.

Sample solution

Heat flow is

$$\hat{Q} = \kappa \frac{A}{L} \Delta T \tag{1}$$

where $\Delta T = 20°C - (-10°C) = 30°C = 30 \text{ K}$.
By (1)

$$\hat{Q} = (0.80 \text{ W/m·K}) \left(\frac{0.90 \text{ m} \times 0.50 \text{m}}{3.0 \times 10^{-3} \text{ m}} \right) (30 \text{ K})$$

$$\boxed{\hat{Q} = 3000 \text{ watt.}} \tag{2}$$
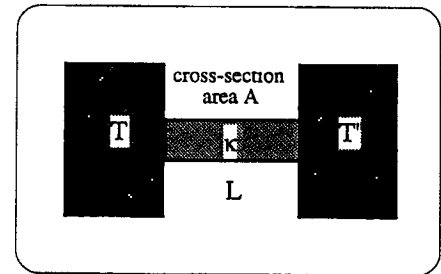

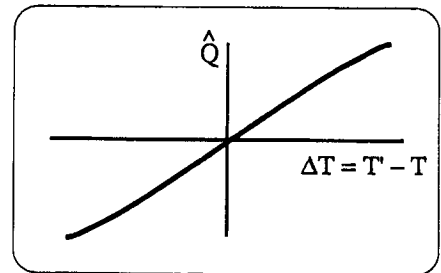**Fig. F-1.** A rod connecting two systems at different temperatures.

Fig. F-2. Graph indicating how the heat flow $\hat{Q}$ from a higher temperature T' to a lower temperature T depends on the temperature difference ΔT.

Substance	κ (W/m K)
Aluminum	205
Copper	385
Silver	406
Brass	109
Steel	50
Glass	0.8
Concrete	0.8
Styrofoam	0.01

Fig. F-3. Thermal conductivities of some substances (at room temperature).

[F-2] *Measuring the thermal conductivity of a material*

To measure the thermal conductivity of a material, an experimenter constructs a box whose walls, made of this material, are 3.2 cm thick and have a total area of 2.1 m^2. The experimenter finds that 150 watt of power must be supplied to an electric heater inside the box in order to maintain the temperature inside the box 55°C above the temperature outside the box. What then is the thermal conductivity of this material? *<a-1>*

[F-3] *Heat flow resulting from window in a door*

A door, 2.20 m high and 0.80 m wide, is made of 3.8 cm thick wood which has a thermal conductivity of 0.050 W/m·K. A small window, 0.50 m high and 0.40 m wide, is then cut into the door. This window is made of 0.40 cm thick glass having a thermal conductivity of 0.80 W/m·K. The door separates the inside of a house, at a temperature of 25°C, from the outside winter air at −10°C.

(a) How much heat per second would flow through the original door (without the window) from the inside of the house to the outside surroundings?

(b) With the window in the door, how much heat per second flows through this window? How much heat flows through the remaining wooden part of the door?

(c) How much larger is the heat flow through the windowed door than the heat flow that would have occurred through the original window-less door? *<a-4>*

G. Summary

Definitions

Heat capacity:

$$C_y = \left(\frac{d'Q}{dT}\right)_y \qquad \text{(where y indicates quantity which is kept constant).}$$

{Describes relation between absorbed heat d'Q and temperature change dT.}

If C is constant, $C = Q/\Delta T$ or $Q = C \, \Delta T$ (for any temperature change ΔT).

Specific heats:

Per mole: $c = C/\nu$ (where ν = number of moles of substance).

Per unit mass: $\underline{c} = C/M$ (where M = mass of substance).

Important knowledge

Heat capacity at constant volume:

$$C_V = \left(\frac{dE}{dT}\right)_V \qquad \text{(since d'Q = dE when now work is done).}$$

Molar specific heat of a monatomic ideal gas:

$$c_V = \frac{3}{2}R = 12.5 \text{ J/(K mole)} \quad \text{(since molar energy } E = \frac{3}{2}RT),$$

Entropy related to heat capacity:

$$S_B - S_A = \int_{T_A}^{T_B} \frac{C \, dT}{T} = C \ln(T_B/T_A) \quad (\textit{if C is constant}).$$

Adiabatic process of an ideal gas:

pV^γ = constant   [where $\gamma = (c_V + R)/c_V$]

Heat flow and thermal conductivity:

$\dot{Q} = \kappa \dfrac{A}{L} \Delta T$ (where κ = thermal conductivity of substance).

New abilities

You should now be able to do the following:

(1) Use the heat capacity to relate a system's temperature change and absorbed heat. Also relate such a heat capacity to specific heats per mole or per unit mass.

(2) Use the thermal interaction between systems to determine temperatures from known heat capacities, or to determine the heat capacity of a system from other known heat capacities and known temperatures.

(3) Use knowledge of a system's heat capacity to determine its entropy change between any two temperatures.

(4) Use the thermal conductivity of a substance to related heat flow to temperature differences.

Problems

[G-1] *Relation between c_V and c_p of an ideal gas*

Suppose that a small amount of heat d'Q is added to one mole of an ideal gas while its pressure p is maintained constant.

(a) How is this absorbed heat related to the change dE of the internal energy of this gas and to the change dV of the volume of the gas? *<h-9>*

(b) How is the energy change dE related to the change dT of the absolute temperature change of the gas? Express your answer in terms of the molar specific heat c_V of the gas *at constant volume.* *<h-11>*

(c) How is the volume change dV related to the change dT of the absolute temperature of the gas? *<h-4>*

(d) How is the molar specific heat c_p of the gas, *at constant pressure*, related to d'Q and dT? By using the answers to the preceding parts of this problem, express this specific heat in terms of the molar specific heat c_V of the gas at constant volume. *<a-26>*

[G-2] *Molar specific heat c_p of some ideal gases*

Exploit the result of the preceding problem to answer the following questions about the molar specific heat c_p of some ideal gases *at constant pressure*. Express your answers in terms of the gas constant R.

(a) What is the molar specific heat c_p of a monatomic ideal gas?

(b) The molar specific heat c_V, at constant volume, of a diatomic gas like nitrogen (N_2) is $\frac{5}{2}R$. What is the molar specific heat c_p of such a diatomic ideal gas maintained at constant pressure? *<a-20>*

[G-3] *Heat capacity C_p and enthalpy change*

The enthalpy H of a system, maintained at a constant pressure p, is defined to be equal to H = E + pV where E is the system's internal energy and V is its volume. Show that the heat capacity C_p of this system at constant pressure is equal to $(dH/dT)_p$.

[G-4] *Final temperature of tea poured into a cup*

A copper cup, having a mass of 120 gram, is at a temperature of 20.0°C and insulated from its surroundings by a wool sleeve. A quarter liter of tea, having a mass of 250 gram, has a temperature of 80.0°C when it is poured into this cup. What is the final temperature of the tea in the cup? *<a-35>*

[G-5] *Temperature change produced by impact of a bullet*

A lead bullet, of mass M_L, travels with a speed v when it hits and partly penetrates an aluminum block. This block, of mass M_A, is attached to a brick wall so that it remains at rest. Initially, both the bullet and the aluminum block are at a temperature of 23°C.

(a) What is the increase of the internal energy of the bullet and aluminum block as a result of the bullet's impact with the block?

(b) What is the increase of the absolute temperature of the block and bullet as a result of the impact? (Assume that negligible heat flows from the block to the wall or its other surroundings.) Express your result in terms of the bullet speed v, the masses of the bullet and block, and the specific heats per unit mass c_L and c_A of lead and aluminum.

(c) The specific heat of lead is 130 J/kg·K and that of aluminum is 910 J/kg·K. Suppose that the mass of the bullet is 0.030 kg and that of the aluminum block is 0.500 kg. What then is the final temperature (in degrees C) of the block and bullet if the bullet strikes the block with a speed of 800. m/s? *<a-15>*

[G-6] *Entropy change in thermal interaction between identical systems*

Problem D-4 considered the thermal interaction between two identical systems, each having a constant heat capacity C. These systems, which had initially absolute temperatures T_1 and T_2, finally reached a common final absolute temperature T_f.

(a) The change of the total entropy of both systems, between the initial situation and the final situation, was already found in Problem D-4. Express this change of the entropy in terms of heat capacity C, the final temperature T_f, and the initial temperature difference $\Delta T = T_2 - T_1$.

(b) Is the final total entropy of the systems larger than or smaller than their initial total entropy? Under what special conditions is the final total entropy the same the initial total entropy? *<a-22>*

[G-7]† *Entropy change in thermal interaction with a heat reservoir*

An object, which has a constant heat capacity C, is initially at an absolute temperature T_A. The object is now brought into thermal contact with a "heat reservoir" at an absolute temperature T_B. (A heat reservoir is a system of such large heat capacity that its temperature remains essentially unchanged irrespective of how much heat it absorbs or gives off.) After equilibrium is reached, the object then attains the same temperature T_B as that of the heat reservoir.

(a) What is the heat absorbed by the object as a result of its thermal interaction with the heat reservoir?

(b) What is the entropy change of the object as a result of this thermal interaction?

(c) What is the entropy change of the heat reservoir as a result of this thermal interaction?

(d) What is the total entropy change ΔS^* of the entire system (object and heat reservoir) as a result of the thermal interaction? Express your answer in terms of the heat capacity C and the ratio $y = T_B/T_A$ of the object's final and initial absolute temperatures.

(e) Draw a graph showing how the total entropy change ΔS^* varies with the temperature ratio y.

(f) For what value of y is the total entropy change minimum? What is the corresponding value of the total entropy change? <h-8>

(g) Under what conditions is the total entropy change of the entire system zero? If it is not zero, is it always positive or always negative? <a-27>

[G-8]† *Entropy changes for various ways of adding heat*

An object has a constant heat capacity C and is initially at an absolute temperature of 300 K. This object can be brought into thermal contact with one or more heat reservoirs

(a) What is the total entropy change of the universe if the object is brought into thermal contact with a heat reservoir at a temperature of 400K and allowed to come to equilibrium with it? (Refer to the results of the preceding problem.)

(b) What is the total entropy change of the universe if the object is brought into thermal contact with a heat reservoir at a temperature of 350 K and allowed to come to equilibrium with it?

(c) What is the total entropy change of the universe if the object is afterwards brought into thermal contact with a heat reservoir at a temperature of 400K and allowed to come to equilibrium with it?

(d) The two successive processes specified in parts *b* and *c* also bring the object from its initial temperature of 300 K to its final temperature of 400 K. What is the total entropy change of the universe as a result of these two successive processes? Is this entropy change larger than or smaller than the entropy change produced in part *a* where the object was brought to its final temperature in a single step? <a-32>

[G-9] *Phase transformations*

The same substance can exist in several different forms or "phases" (e.g., in the form of a solid, a liquid, or a gas). The substance changes from one form to another at some definite temperature and remains at this temperature while changing its form. But some heat (called "heat of transformation" or "latent heat") must be added to the substance to melt it so as to change it from the solid to the liquid form. Similarly, some latent heat must be added to vaporize the substance so as to change it from the liquid form to the gaseous form (or so as to change if from the solid to the gaseous form).

As indicated in Fig. G-1, water changes from its solid form (called "ice") to its liquid form at a temperature of 0°C. The heat of transformation required to effect this change is 3.33×10^5 J/kg (i.e., 3.33×10^5 joule of heat must be added to melt every kilogram of ice).

Suppose that a 500-watt electric heater is used to melt ice. How much ice would it melt after 5.0 minutes? <a-11>

Substance	Melting point (K)	Heat of fusion (kJ/kg)	Boiling point (K)	Heat of vaporization (kJ/kg)
Hydrogen	14.0	58.6	20.3	452
Oxygen	54.8	13.8	90.2	213
Mercury	234	11.3	630	296
Water	273	333	373	2256
Lead	601	24.7	2013	858
Silver	1235	105	2485	2336

Fig. G-1. Heats of transformation of some common substances at atmospheric pressure. (The melting point is the temperature at which the substance changes from solid to liquid. The boiling point is the temperature at which is changes from liquid to gas.)

[G-10] *Cooling tea with ice*

What mass of ice, originally taken from a freezer at a temperature of −15°C, must be added to 0.25 kg of tea, originally at a temperature of 80°C, so as to end up with cold tea at a temperature of 10°C? (The specific heat of tea is the same as that of water, i.e., 4190 J/kg K. The specific heat of ice is 2000 J/kg K and its latent heat of melting is 3.33×10^5 J/kg.)

The following illustrates the solution of this problem.

Sample solution

<u>Situation:</u>

Initially: Tea: Mass = 0.25 kg. τ = 80°C.

Ice: Mass = M. τ = –15°C.

Finally: Tea (and melted ice) at 10°C.

<u>Goal:</u> M = ?

Apply energy law to tea + ice, between initial and final states:

$$(\Delta E)_{tea} + (\Delta E)_{ice} = 0. \tag{1}$$

Tea.

Initial state: Tea at 80°C.

⇓ c = 4190 J/kg K.

ΔE = (0.25 kg) (4190 J/kg K) [(10°C – 90°C) (K/•C)]

ΔE = –7.33 × 10^4 J. $\tag{2}$

Final state: Tea at 10°C.

Ice.

Initial state: Ice at –15°C.

⇓   Warming ice. c = 2000 J/kg K

ΔE = M (2000 J/kg K) {[0°C – (–15°C)] (K/°C)}

ΔE = 3.00 × 10^4 M (J/kg) $\tag{3}$

Intermediate state #1: Ice at 0°C.

⇓   Melting ice at 0°C. Latent heat = 3.33 × 10^5 J/kg.

ΔE = M (3.33 × 10^5 J/kg)

ΔE = (3.33 × 10^5) M (J/kg). $\tag{4}$

Intermediate state #2: Melted ice (i.e., water) at 0°C.

⇓   Warming resulting water. c = 4190 J/kg K.

ΔE = M (4190 J/kg K) [(10°C – 0°C) (K/°C)]

ΔE = 4.19 × 10^4 M (J/kg). $\tag{5}$

Final state: Melted ice (i.e., water) at 10°C.

Put results (2) through (5) into (1):

[–7.33 × 10^4 J] + [3.00 × 10^4 + 3.33 × 10^5 + 4.19 × 10^4] M (J/kg) = 0

–7.33 × 10^4 J + 40.5 × 10^4 M (J/kg) = 0

7.33 × 10^4 kg = 40.5 × 10^4 M

$$\boxed{M = 0.18 \text{ kg}} \tag{6}$$

[G-11] *Water frozen by an ice cube*

An ice cube, having a mass of 75 gram, is taken from a freezer at temperature of –15°C and is dropped into a glass of water at a temperature of 0°C. How many grams of that water will freeze as a result? (Data about the properties of ice and water are given in Figs. B-1 and G-1.) <a-24>

[G-12] *Temperature reached by water cooled by an ice cube*

A copper can, having a mass of 0.750 kg and filled with 0.200 kg of water at a temperature of 20.0°C, is thermally well insulated from its environment. A person now puts into the water 0.030 kg of ice at a temperature of 0.0°C.

(a) What will be the final temperature of the water after the ice has melted and final equilibrium has been reached?

(b) How much work must afterwards be done to rotate a stirring rod, immersed in the water, to bring the copper can and its contents back to the temperature of 20.0°C? (The heat capacity of the stirring rod is negligible.) <a-28>

[G-13]† *Cooling soda with ice*

A person pours 100 gram of soda (which is essentially water) into a thermally insulated copper mug having a mass of 275 gram. The soda and mug are initially at a temperature of 20°C. The person then drops into the soda 50 grams of ice taken from a freezer at a temperature of –15°C.

(a) What is the final temperature reached by the soda (before heat losses to the surrounding room become significant)?

(b) Has all the ice then melted? If not, how much ice remains in the mug? <h-10> <a-29>

[G-14] *Effect of temperature on energy of air in a room*

Air consists of the diatomic molecules nitrogen and oxygen. The average energy of such a diatomic molecule, at an absolute temperature T, is (5/2)kT.

(a) Suppose that the temperature of the air in a room changes from 17°C to 37°C while the pressure of the air remains at atmospheric pressure $(1.01 \times 10^5 \text{ N/m}^2)$. By what amount (in joules) does the average energy of an air molecule then change?

(b) Does the total energy of the air molecules in the room increase, decrease, or remain the same? Why?

(c) Does the number of air molecules in the room increase, decrease, or remain the same? <a-44>

[G-15] *Different gas processes between the same temperatures*

The specific heat of one mole of an ideal diatomic gas is $\frac{5}{2}$ R (where R is the gas constant). What is the work done on this gas, the heat absorbed by it, and the change of its internal energy in each of the following processes if each raises the absolute temperature of the gas from T to T'?

(a) A process in which the volume of the gas remains constant?

(b) A process in which the pressure of the gas remains constant?

(c) An adiabatic process? <a-40>

[G-16] *Temperatures in a cyclic gas process*

Fig. G-2 illustrates a cyclic quasi-static process *ABCA* of a monatomic ideal gas.

(a) The volume of the gas remains constant during the process *AB*. Is the absolute temperature T_B of the gas in the state *B* larger than, smaller than, or equal to its absolute temperature in the state *A*? Why?

(b) The gas is thermally insulated during the process *BC*. Is the absolute temperature T_C of the gas in the state *C* larger than, smaller than, or equal to its absolute temperature in the state *B*? Why?

(c) The pressure of the gas remains constant during the process *CA*. Is the absolute temperature T_A of the gas in the state *A* larger than, smaller than, or equal to its absolute temperature in the state *C*? Why?

(d) What is the temperature T_B? Express your answer in terms of T_A and the volumes V_A and V_C of the gas in the states *A* and *C*.

(e) What is the temperature T_C? Express your answer in terms of the same quantities. <h-12> <a-46>
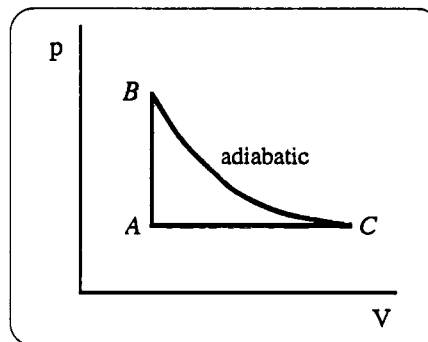

Fig. G-2. Cyclic process of a monatomic ideal gas.

[G-17] *Adiabatic expansion of oxygen gas*

Oxygen gas, which is diatomic, has a molar specific heat equal to $\frac{5}{2}R$. At a temperature of 35°C, the volume occupied by 8.00×10^{-2} moles of this gas is 1.00×10^{-3} m^3. The gas is now expanded adiabatically until it reaches a temperature of 12°C.

(a) What is the original pressure of the gas?
(b) During the expansion of the gas, what is the work done on the gas, what is the heat absorbed by the gas, and what is the change of internal energy of the gas?
(c) What are the final volume and pressure of the gas? <*a-37*>

[G-18] *Expansion of helium gas*

One mole of helium gas, initially at an absolute temperature T_A, has a volume V_A. As illustrated by the graph in Fig. G-§, the gas is now expanded, at constant pressure, until its volume is $2V_A$. The gas is then further adiabatically expanded until its temperature reaches the same value T_A as it had originally.

(a) What is the work done on the gas, and the heat absorbed by the gas, during this entire process?
(b) What is the final volume of the gas? <*a-43*>

[G-19] *Cyclic process of a monatomic gas*

Fig. G-3 shows how the pressure p of one mole of a monatomic ideal gas varies with its volume V during the cyclic process *ABCA*. Express the answers to the following questions in terms of the gas constant R and the absolute temperatures T_A, T_B, and T_C of the gas in the states *A*, *B*, and *C*.

Fig. G-3. Cyclic process of a monatomic gas.

(a) What is the work done on the gas, the heat absorbed by it, and the change of its internal energy in the process *AB* during which the volume of the gas remains constant?
(b) What is the work done on the gas, the heat absorbed by it, and the change of its internal energy in the adiabatic process *BC*?
(c) What is the work done on the gas, the heat absorbed by it, and the change of its internal energy in the process *CA* during which the pressure of the gas remains constant?
(d) Suppose that the temperatures are $T_A = 300.$ K, $T_B = 650.$ K, and $T_C = 450.$ K. What then are the numerical answers to each of the questions posed in parts *a*, *b*, and *c*?
(e) What is the numerical value of the total work done on the gas during the entire cyclic process *ABCA*? What is the numerical value of the total heat absorbed by the gas during this entire process? <*a-48*>

[G-20] *Pressures and volumes in the preceding process*

Suppose that the preceding mole of monatomic gas starts in state *A* at atmospheric pressure (1.013×10^5 N/m^2).

(a) What then is the volume of the gas in this state *A*?
(b) What are the volume and pressure of the gas in the state *B*?
(c) What are the volume and pressure of the gas in the state *C*? <*a-41*>

[G-21] *Size dependence of heat conduction*

To examine in greater detail how the heat flow through a rod depends on its size, consider first a small block of length L_0 and cross-sectional area A_0. The heat $\hat{Q}_0$ flowing through this block is proportional to the temperature difference $(\Delta T)_0$ across its ends so that

$$\hat{Q}_0 = B\,(\Delta T)_0 \tag{1}$$

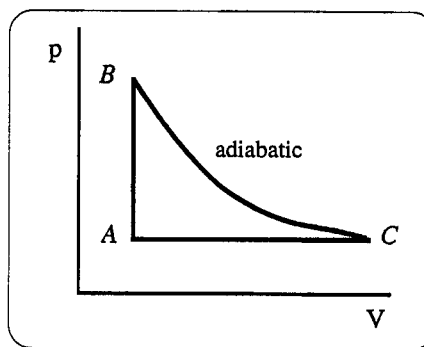where B is some constant characterizing the small block.

As indicated in Fig. G-4, the rod can be imagined to consist of many identical such small blocks placed end-to-end and side-by-side.

(a) If the length of the rod is L, there are L/L_0 such blocks placed end-to-end. What is the temperature difference $(\Delta T)_0$ across a single such block if the temperature difference across the entire length of the rod is ΔT?

(b) If the cross-sectional area of the rod is A, there are A/A_0 such blocks placed side-by-side. What is the heat flow $\hat{Q}_0$ through a single such block if the heat flow through all the side-by-side blocks is $\hat{Q}$?

(c) Use the preceding relation to express the relation (1) in terms of the quantities characterizing the entire rod (i.e., the quantities $\hat{Q}$, ΔT, L, and A). According to this result, how does the heat flow $\hat{Q}$ through the rod depend on its length L? How does it depend on its cross-sectional area A?·

(d) Are the preceding results consistent with the relation (G-2)? Suppose that the rod consists of a substance having a thermal conductivity κ. Express this thermal conductivity in terms of the quantities B, L_0, and A_0 characterizing a small block of this substance. *<a-14>*

Fig. G-4. A rod consisting of many identical small blocks.

[G-22] *Power needed to maintain an oven temperature*

The electric oven of a kitchen range is insulated with a 3.5 cm thick layer of fiberglass and has a total wall area of 1.30 m^2. The thermal conductivity of the fiberglass is 0.040 W/m·K. How much electric power must be supplied to the heating element in this oven so as to maintain its inside temperature at 220°C when the outside room temperature is 20°C? *<a-36>*

[G-23] *Heat conduction through two connected bars*

Two bars, of the same cross-sectional area, are joined together end-to-end as illustrated in Fig. G-5. The first bar has a length L_1 and is made of a material having a thermal conductivity κ_1. The second bar has a length L_2 and is made of a material having a thermal conductivity κ_2. The free end of the first bar is in contact with ice and the free end of the second bar is in contact with warm water. (The bars are, however, wrapped with thermally insulating material so that negligible heat flows into them through their sides.)
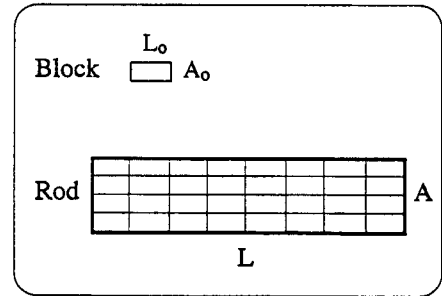
In a steady state, the temperature at any point of the bars remains unchanged. No *net* heat is then absorbed by any small region within in the bars, i.e., the heat entering any such region from one side must be equal to the heat leaving it from the other side. In particular, this means that the heat flowing through the first bar must then be equal to the heat flowing through the second bar (since the temperature near their junction would otherwise change).

In such a steady state, what is the ratio $(\Delta T)_2/(\Delta T)_1$ of the temperature difference $(\Delta T)_2$ existing across the ends of the second bar compared to the temperature difference $(\Delta T)_1$ existing across the ends of the first bar? Express your answer in terms of the lengths of the two bars and in terms of their thermal conductivities. *<a-9>*
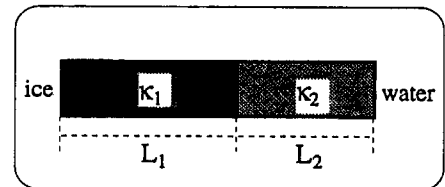

Fig. G-5. Heat conduction through two connected bars.

[G-24] *Heat flow through an insulated brick wall*

The wall of a house is 10 cm thick and consists of bricks having a thermal conductivity of 0.60 W/m·K. The inside of the house is maintained at a temperature of 25°C and the outside air temperature is –15°C.

(a) How much heat per second leaves the inside of the house as a result of conduction through every square meter of the wall?

(b) Suppose that the inside surface of the brick wall is covered with a 2 cm thick layer of rock wool which has a thermal conductivity of 0.04 W/m·K. If the inside of the house is again maintained at a temperature of 25°C and the outside air temperature is again – 15°C, what is the temperature at the boundary where the rock wool is joined to the brick wall? *<h-1>*

(c) Under these conditions, how much heat per second leaves the inside of the house as a result of conduction through every square meter of the wall? <a-16>

[G-25]‡ *Time needed to form an ice layer on a lake*

A sheet of ice forms at the top of a lake when the air temperature at the surface of the lake is lower than the 0°C temperature at which water freezes. The temperature of the water below the ice sheet remains near 0°C. But when a layer of water freezes at the bottom of the ice sheet, the heat liberated in the process flows through the ice sheet to the cold air above it.

Suppose that the temperature difference between the bottom and top of the ice sheet is ΔT. The heat given off when water is transformed into ice is $L = 3.33 \times 10^5$ J/kg, the density of ice is $\rho = 0.92 \times 10^3$ kg/m^3, and the thermal conductivity of ice is $\kappa = 1.6$ W/m K.

(a) Derive a relation showing how the thickness x of the ice sheet depends on the elapsed time t since the ice started to form. (Besides x and t, this relation should involve the constants $\Delta T, L, \rho$, and κ.) <h-3>

(b) Suppose that a time T is required to form a 0.10 m thick sheet of ice. How long a time would then be required to form a 0.20 m thick sheet of ice?

(c) Suppose that the temperature at the top of the lake is −10°C. How many hours would then be required to form a sheet of ice 0.10 m thick? <a-7>