2   Work and Heat

A. Macroscopic work on a system
B. Thermodynamic energy law
C. Measurement of internal energy and heat
D. Applications of the thermodynamic energy law
E. Summary

A macroscopic system can interact with other macroscopic systems in its environment. How can such interactions be usefully described in ways that are macroscopically measurable? How do they affect the energy of the system and what are some of the resulting implications? The following sections deal with these basic questions.

A. Macroseopic work on a system

Internal energy of a system. Consider any macroscopic system at rest, e.g., the gas in the cylinder illustrated in Fig. A-1 or the stretched rubber band in Fig. A-2. Although the center of mass of such a system is at rest, the internal energy E of the system (i.e, the sum of the kinetic energies and of the potential energies of the all the atoms in the system) can be quite large.

For example, in Fig. A-1 the molecules of the gas (and thus also the atoms in these molecules) move with large speeds throughout the enclosing cylinder and thus have large kinetic energies. Similarly, in Fig. A-2 the molecules in the rubber band vibrate slightly around their fixed positions. Thus they have kinetic energy due to their vibrational motion and also potential energy due to their interactions with neighboring molecules.

Changing internal energy by macroscopic work. The energy of any such system can change if it interacts with other systems. One way of changing this energy is by doing some macroscopic work W on the system, i.e., by causing some macroscopic displacement in the presence of some macroscopic force. For instance, in Fig. A-1 the piston can be moved through some readily visible distance in the presence of the applied force F opposing the pressure force caused by the gas. If the piston is moved inward along the direction of this force (so that the gas is compressed), some positive macroscopic work is then done on the gas. Conversely, if the piston is moved outward (so that the gas is expanded), some negative macroscopic work is done on the gas.

We consider a system at rest if its center of mass remains at rest relative to us on the surface of the earth (or more precisely, relative to some inertial frame).
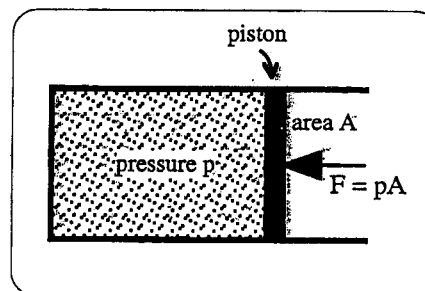

Fig. A-1. A gas in a cylinder closed by a movable piston. (The applied force F keeps the cylinder stationary.)
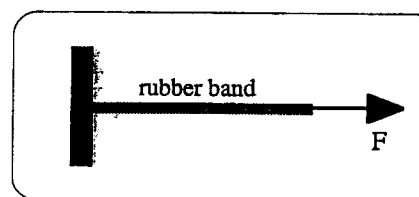

Fig. A-2. A rubber band stretched by an applied force F.

Work done in changing the volume of a fluid

Pressure work due to a small volume change. The macroscopic work can readily be calculated from a knowledge of macroscopically measurable quantities. For example, Fig. A-3 illustrates a fluid (gas or liquid) inside a cylinder closed by a movable piston of area A. The fluid, in equilibrium at a pressure p, exerts on the piston a force pA to the right. The piston is maintained at rest by an applied force of magnitude F = pA directed to the left.

Suppose that the piston is now moved to the right by a distance s which is so very small that the pressure of the fluid remains essentially equal to p. Assume also that this is done so slowly that the fluid remains essentially in equilibrium., e.g., so that the pressure has always the same value at all points in the fluid. During this very small displacement of the piston the magnitude F of the applied force remains then equal to pA. Hence the very small work d'W done by this force on the system (consisting of fluid and piston) is equal to

$$d'W = - F s = - pA \, s \, . \tag{A-1}$$

Here the minus sign indicates that this work is negative if s is positive (i.e., if the piston's displacement is outward, opposite to the direction of the applied force).

The very small displacement of the piston results in a corrrespondingly small volume change dV of the system. As is apparent from Fig. A-3, this volume change is equal to the piston's cross-sectional area multiplied by the distance through which it moves. Thus dV = As and the small work (A-1) done on the system is simply equal to

$$\boxed{d'W = - p \, dV. } \tag{A-2}$$

This work is negative if dV is positive (i.e., if the gas expands so that its volume increases and the applied force is opposite to the piston's outward motion). Conversely, this work is positive if dV is negative (i.e., if the gas is compressed so that its volume decreases and the applied force is along to the piston's inward motion).

Pressure work due to any quasi-static volume change. Consider a process in which the volume of a fluid is changed by an appreciably amount, but so very slowly as to be quasi-static.

Def: | **Quasi-static process:** A process occurring so slowly that the system of interest remains always very nearly in equilibrium. | (A-3)

At any instant, the pressure in the fluid has then some particular value p which the same throughout the fluid. According to (A-2), a small amount of work d'W = - p dV is then done on the fluid when its volume is changed by a very small amount dV.

Suppose now that the volume of the fluid is changed quasi-statically by successive small amounts, starting from some initial volume V_A (when the fluid has some pressure p_A) and ending with some appreciably different final volume
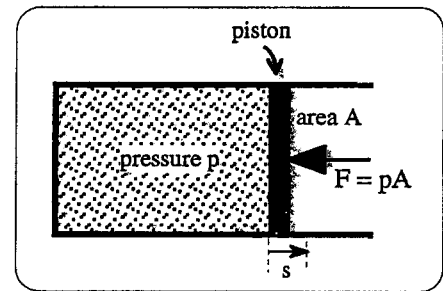

Fig. A-3. Small volume change of a fluid, with resulting work done on the fluid.

Although the distance s may be infinitesimally small compared to macroscopic dimensions, it is still supposed much larger than atomic size

The symbol d' (with the prime appended to the letter d) is used to indicate any infinitesimally small quantity (which needs not necessarily be equal to some infinitesimal difference).

Imagine that the piston is very thin. Then (A-2) is just the work done on the fluid by outside forces when the bounding surface of the fluid moves and its volume changes correspondingly.

V_B (when the fluid has some pressure p_B). The total work W done on the system is then merely the sum of the successive small works done on it. Thus

$$W = \int d'W = -\int p \, dV \qquad (A\text{-}4)$$

where the symbol $\int$ denotes the sum (or *integral*) of all the successive small works, starting with the initial volume V_A and ending with the final volume V_B.

Graphical representations. One often knows the pressure of a fluid when it is in equilibrium under specifieid conditions and has a known volume. For example, this information can be summarized by a graph, like that in Fig. A-4, which indicates the relation between the pressure p and volume V of such a fluid.

Fig. A-4. Graph showing the relation between the pressure p and volume V of a fluid.

Suppose that the pressure is of the fluid is p and that its volume changes by an infinitesimal amount dV. By (A-1), the *magnitude* |d'W| of the work done on the fluid is then p |dV| (i.e., it is equal to the pressure multiplied by the magnitude of the infinitesimal volume change). On a graph like that in Fig. A-4, this work is thus equal to the area of the indicated rectangle having a height p and a very narrow width equal to the magnitude of the infinitesimal volume change dV.

Remember that the *magnitude* of a numerical quantity is the positive number obtained by ignoring its sign.

Suppose now that the volume of the fluid is changed quasi-statically from an initial value V_A to a final value V_B. (As indicated in Fig. A-5, the fluid then goes from the macrostate A, where its volume is V_A and its pressure is p_A, to the macrostate B where its volume is V_B and its pressure is p_B.) The magnitude of the total work done on the fluid in this process (i.e., the sum of the infinitesimal works done during all the successive infinitesimal volume changes) is then equal to the sum of the areas of all the successive infinitesimal rectangles in Fig. A-5. Thus it is just equal to the indicated total area under the graph in Fig. A-5.

Fig. A-5. Fluid going between two macrostates A and B. (The magnitude of the work done in this process is equal to the indicated area under the graph.)

Problems

[A-1] *Units of work*

The unit of work in the SI system (the international system of units) is called *joule* (abbreviated as J). We also know that, by virtue of its definition, work must have the units of a force multiplied by a distance. Use this knowledge to express the unit joule in terms of the basic SI units of length, time, and mass [i.e., the units meter (m), second (s), and kilogram (kg)]. <h-3> <a-7>

The unit *joule* is named after James Prescott Joule (1818-1889), the English physicist whose careful experiments led to a quantitative understanding of the relationship between heat and energy.

[A-2] *Work done in slightly stretching a rubber band*

The rubber band illustrated in Fig. A-2 has a length L and is kept stretched by the indicated applied force having a magnitude F. Suppose that the rubber band is now slightly stretched some more so that its length increases by the infinitesimal amount dL.

(a) What then is the work done on the rubber band by the applied force?
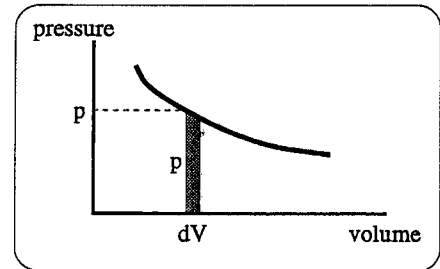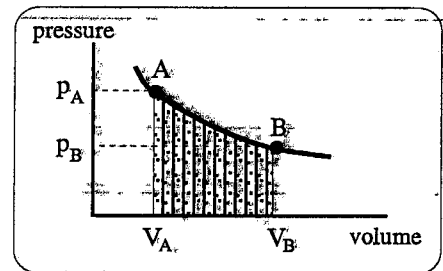(b) Is this work positive or negative? <a-5>

[A-3] *Work done during a slight expansion of a gas*

Argon gas is enclosed in a cylinder closed by a piston. The gas is in equilibrium at a pressure of $1.60 \times 10^5 \, N/m^2$ and occupies a volume of $0.0425 \, m^3$.

(a) How much work must be done by an outside force applied to the cylinder so as to expand the gas to a volume of $0.0428 \, m^3$? (The pressure of the gas remains essentially unchanged during this compression.)

(b) Is this work positive or negative? *<a-3>*

[A-4] *Comparing work done in two different processes*

A gas is initially in a macrostate A where its volume is V_A and its pressure is p_A. Graph 1 in Fig. A-6 indicates how the gas pressure p varies when the gas is kept at room temperature and slowly compressed until its volume V becomes equal to V_B. Graph 2 in Fig. A-6 indicates how the pressure of the gas varies when it again starts in state A, but is thermally insulated from its surroundings (so that its temperature can change) and slowly compressed to the same smaller volume V_B.

In which of the two processes, that indicated by graph 1 or by graph 2, is more macroscopic work done on the gas during its compression? Or are the works the same? Explain your reasoning. *<a-9>*

Fig. A-6. Different compressions of a gas.

[A-5] *Work done in a cyclic process*

A gas is initially in the macrostate A where its volume is $6.0 \times 10^{-3} \, m^3$ and its pressure is $5.0 \times 10^5 \, N/m^2$.

The volume V and pressure p of the gas are now quasi-statically changed in the way illustrated by the graph in Fig. A-7. (The temperature of the gas also changes during these processes.). (a) The volume of the gas is first changed, at constant pressure, until the gas reaches the macrostate B where its volume is $2.0 \times 10^{-3} \, m^3$. (b) The pressure of the gas is then changed, at constant volume, until the gas reaches the macrostate C where its pressure is $8.0 \times 10^5 \, N/m^2$. (c) The volume of the gas is then changed, at constant pressure, until the gas reaches the macrostate D where its volume is again $6.0 \times 10^{-3} \, m^3$. (d) Finally, the pressure of the gas is changed, at constant volume, until the gas reaches again its initial macrostate A where its pressure is $5.0 \times 10^5 \, N/m^2$.

(a) What is the work (magnitude and sign) done on the gas in the first process $A \rightarrow B$?

(b) What is this work in the second process $B \rightarrow C$?

(c) What is this work in the third process $C \rightarrow D$?

(d) What is this work in the last process $D \rightarrow A$?

(e) What is the total work (magnitude and sign) done on the gas during the entire process ABCDA? (Such a process is called *cyclic* since the system returns to its initial state.) *<a-12>*

(f) Show that the magnitude of this total work is equal to the area of the rectangle ABCD indicated in the graph of Fig. A-7.

(g) Calculate the area of this rectangle. Verify that it is equal to the magnitude of the work previously calculated in part *e*.

(h) Suppose that the cyclic process is carried out in the reverse order (i.e., in the order ADCBA). How does the total work done on the gas in this reverse process differ from the total work done on the gas in the original process? *<a-1>*
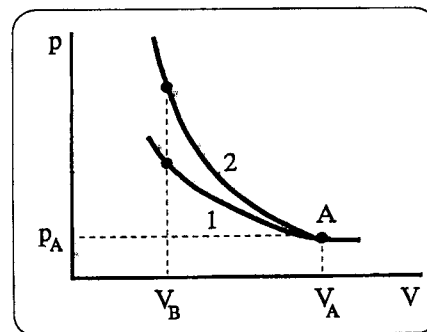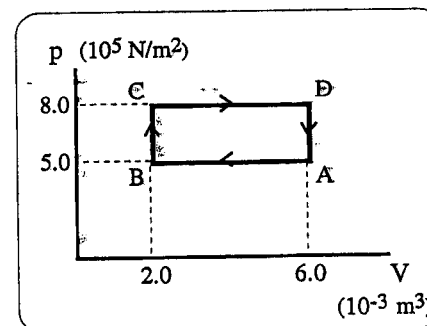

Fig. A-7. Cyclic process of a gas.

C. Measurement of internal energy and heat

We obtained the thermodynamic energy law (B-6) by decomposing the entire work done on a macroscopic system into macroscopic and non-macroscopic parts, i.e., into work and heat. But is this decomposition really meaningful from a macroscopic point of view, i.e., can all the quantities appearing in the thermodynamic energy law actually be determined by large-scale measurements?

As already discussed in Sec. A, macroscopic work can clearly be measured by observations of macroscopic displacements and macroscopic forces. But what about the change ΔE of internal energy (i.e., the change in the sum of the kinetic and potential energies of all the atomic particles in the system)? And what about the absorbed heat Q which is entirely non-macroscopic, due solely to work done during interactions between individual atomic particles?

At first sight one might well believe that there is no way that purely macroscopic measurements could be used to determine these quantities which are so directly related to atomic particles. But this belief is false, i.e., these quantities can actually be determined by large-scale measurements provided that one goes about the task in the right way. The following simple analogy indicates how this can be done and helps to clarify many of the issues involved in applying the thermodynamic energy law.

Analogy to water in a lake

Processes affecting the water in a lake. Consider a lake, like that schematically illustrated in Fig. C-1. The state of such a lake may be specified by the amount A of water in it. This amount of water may change because of the following two kinds of processes.

Fig. C-1. A lake interacting with its environment. (Water can enter or leave the lake as a result of flow or condensation.)

* **Flow.** One macroscopically visible process occurs when some amount F of water flows into the lake from some stream. (A positive flow F indicates water flowing into the lake. Conversely, a negative flow F indicates water flowing out of the lake.)

* **Condensation.** Another process, not macroscopic and thus not readily visible, occurs when some amount C of water condenses into the lake from the surrounding air. (A positive condensation C indicates water condensing into the lake from the air. Conversely, a negative condensation C indicates water evaporating from the lake into the air.)

The change ΔA of the amount of water in the lake is thus generally due to both macroscopic and non-macroscopic processes, i.e., it is due both to flow F and condensation C. Correspondingly, one can write that

$$\Delta A = F + C. \qquad (C-1)$$

Analogy to the thermodynamic energy law

The relation (C-1) is analogous to the thermodynamic energy law (A-6). As indicated in Fig. C-2, the amount of water A in the lake
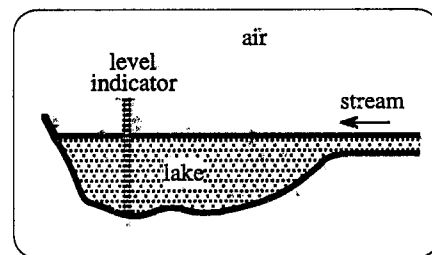
corresponds to the internal energy E of the macroscopic system. The macroscopically visible flow F of water into the lake corresponds to the macroscopic work W done on the system. Finally, the macroscopically invisible condensation C of water into the lake corresponds to the heat Q absorbed by the system.

How can one measure the quantities describing the lake? Since the water flowing in the stream is readily visible, the flow F of water in or out of the lake can readily be measured. But how can one measure the other quantities appearing in the relation (C-1), i.e., the change ΔA in the amount of water in the lake and the condensation C of water into the lake? The following comments indicate how these quantities can be determined.

Level indicator as indicator of amount. Suppose that the flow of water into the lake is stopped (e.g., by means of a damn at the mouth of the stream). Then the amount of water in the lake can nevertheless change because of condensation, and this change can be detected by any indicator of the level of water in the lake. For example, such a level indicator might be a rod, fastened to the bottom of the lake and protruding through the water surface, so that changes of the water level along the rod can be readily noted. Similarly, the water level might be indicated by the observed height of water at the edge of the lake.

Measuring changes of water amount. As the amount of water in the lake changes, the indicated water level changes by a corresponding amount. But how can one quantitatively determine the actual change ΔA in the amount of water corresponding to any indicated change of water level?

To do this, one can eliminate all water condensation (e.g., by covering the lake by a large tarpaulin) and let water flow into the lake through the stream. Then any change ΔA in the amount of water in the lake is just equal to the flow F of water and this flow can readily be measured. Thus one can determine the actual relation between any indicated change of water level and the corresponding change ΔA in the amount of water in the lake.

Amount of water in a lake	Internal energy of a system
A = amount of water.	E = internal energy of system.
F = flow. (Flow is positive for inflow, negative for outflow.)	W = work done on system. (Negative work is work done by system.)
C = condensation. (Negative condensation is evaporation.)	Q = heat absorbed by system. (Negative heat is heat given off.)
$$\Delta A = F + C$$	$$\Delta E = W + Q$$
* The macroscopic flow F can readily be measured.	* Macroscopic W can readily be measured.
* Level indicator can be used to indicate changes of amount.	* Thermometer can be used to indicate changes of internal energy.
* Level indication can be related to change ΔA of amount by measuring flow when condensation is eliminated.	* Thermometer indication can be related to change ΔE of internal energy by measuring work when heat is eliminated.
* Can then measure condensation C since $C = \Delta A - F$.	* Can then measure heat Q since $Q = \Delta E - W$.

Fig. C-2. Analogy between the amount of water in a lake and the internal energy of a macrocopic system.

Measuring condensation. Consider now the general situation where both flow and condensation can occur. According to (C-1), the condensation is then equal to

$$C = \Delta A - F, \qquad (C\text{-}2)$$

i.e., it is just equal to the change in the amount of lake water if one subtracts the inflow of water from the stream. But the change ΔA in the amount of water can be determined from the indicated change of the water level (since we already know how such a change is related to the change in amount). Furthermore, the flow F is readily measured. According to (C-2), the condensation C can then be obtained by a simple subtraction.

Application to the energy of a macroscopic system

The preceding analogy suggests how one can similarly determine the change of internal energy of a macroscopic system and the heat absorbed by it. The following paragraphs (and Fig. C-2) outline how this can be done.

Thermometer as indicator of internal energy. Suppose that no macroscopic work is done on a system (e.g., that the volume of the system is kept fixed). Then the internal energy of the system can nevertheless change because heat can be absorbed by the system, and this change can be detected by some indicator called a *thermometer*.

Def: | *Thermometer:* Any relatively small device characterized by a single macroscopic quantity (called its indicated "temperature") which changes when the device absorbs heat from some other system. | (C-3)

The following are some examples of thermometers: (a) A very familiar thermometer consists of a thin glass tube filled with mercury or some other liquid. The length of the column of liquid in this tube is the temperature indicated by this particular thermometer. (b) A *gas thermometer* consists of a small glass bulb filled with gas. The pressure of this gas is the temperature indicated by this thermometer. (c) Another kind of thermometer consists of a metal bars that rotate a pointer when they change in length. The position of the pointer on a dial is the temperature indicated by this thermometer. (d) Still other thermometers indicate their temperatures by changing electric quantities (such as electric resistance or voltage).

Any such thermometer can be brought into thermal contact with the system of interest and allowed to remain there until equilibrium is reached. The indicated temperature (e.g., the length of the column of mercury in the thermometer) provides then some information about the state of the system. For example, suppose that no work is done on the system, but that the indicated temperature changes. Then one would know that the internal energy of the system changes (i.e., that it must be absorbing heat from some other systems).

Note that the temperature thus defined merely characterizes the particular thermometer being used. We shall see later how it is possible to introduce more universal temperature scales (e.g., the Fahrenheit or Celsius scales) common to different kinds of thermometers.

Measuring changes of internal energy. As the internal energy of a system changes, the temperature indicated by a thermometer changes by a corresponding amount. But how can one quantitatively determine the actual change ΔE in the amount of internal energy corresponding to any change of the indicated temperature?

To do this, one can eliminate all heat transfer to the system (e.g., by wrapping the system with thermally insulating material) and merely do macroscopic work on the system. Then any change ΔE of the system's internal energy is just equal to the macroscopic work W done on the system and this work can be readily measured. Thus one can determine the actual relation between any indicated change of temperature and the corresponding change ΔE in the internal energy of the system.

Measuring heat. Consider now the general situation where work can be done on the system and heat can also be absorbed by it. According to the thermodynamic energy law (B-6), the heat absorbed by the system is then equal to

$$Q = \Delta E - W, \tag{C-4}$$

i.e., it is just equal to the change of its internal energy if one subtracts the work done on the system. But the change ΔE in the system's internal energy can be found from the change of the indicated temperature (since we have determined know how such a change is related to the change of internal energy). Furthermore, the work W is readily measured. According to (C-4), the heat Q can then be obtained by a simple subtraction.

Problems

[C-1] *Increasing the internal energy of a liquid by stirring*

Fig. C-3 shows a paddle wheel immersed in a liquid enclosed within a container. The container is wrapped by a thermally insulating material (e.g., glass wool) so that negligible heat can pass through its walls. The paddle wheel is connected to a weight by a thing string passing over some pulleys. As the weight descends, work is done to rotate the paddle wheel and thus to stir the liquid (whose volume remains unchanged).

Suppose that the weight has a mass of 0.50 kg and that it descends with constant speed through a distance of 1.6 m. What then is the increase of the internal energy of the system consisting of the liquid, the paddle wheel, and the container? <h-8> <a-19>

Fig. C-3. A paddle wheel rotated inside a liquid.

[C-2] *Measuring the heat absorbed by a system*

A jar contains water, a thermometer, and a stirrer which can be rotated by an electric motor. The jar is wrapped by thermally insulating material so that negligible heat can pass through its walls. The initial temperature reading of the thermometer indicates that the column of mercury in the glass tube of the thermometer has a length of 7.1 cm.

After the stirrer is rotated for 2 minutes (during which 600 J of work is done on the stirrer by the motor), the length of the thermometer column is observed to be 7.3 cm. After the stirrer is rotated for another 2 minutes (during which 600 J of work is again done on the stirrer by the motor), the length of the thermometer column is observed to be 7.5 cm. After the stirrer is rotated for another 2 minutes (during which 600 J of work is again done on the stirrer by the motor),
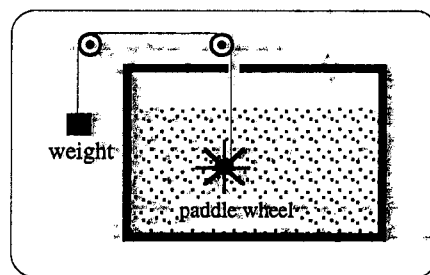
the length of the thermometer column is observed to be 7.7 cm. After that, the insulating material is removed from the jar and the jar is left sitting on a desk.

(a) The next morning, the length of the thermometer column is observed to be 7.3 cm. At noon, it is observed to be 7.7 cm. Hence one concludes that heat has been absorbed by the system (consisting of the jar, water, and stirrer) from the surrounding air in the room. How much heat has thus been absorbed by the system between morning and noon? Is this heat positive or negative?

(b) In the evening the length of the thermometer column is observed to be 7.2 cm. How much heat has been absorbed by the system from the room during the time between noon and evening? Is this heat positive or negative? <a-8>

[C-3] *Measuring the change of internal energy between two macrostates*

A system consists of a gas contained in a cylinder closed by a movable piston. A stirrer is immersed in the gas and the whole system is thermally insulated to that no heat can enter or leave the system. In the macrostate A, the volume of the gas is V_A and its pressure is p_A. In the macrostate B, the gas has a volume V_B larger than V_A and has a pressure p_B larger than p_A. One wants to measure the difference $E_B - E_A$ of the internal energy of this system between these two states.

One may do this in the following way: (1) One may first move the piston so as to increase the volume of the gas from its volume V_A to the desired final volume V_B. During this process some work W_{pis} is done on the system by the piston and the gas ends up in some macrostate where its pressure has some value p' smaller than p_A. (2) Then one may keep the piston in its position (so that the volume of the gas remains equal to V_B) but do some work W_{stir} to rotate the stirrer and thus to increase the pressure of the gas to its desired final value p_B.

(a) Is the work W_{pis} done on the system by the piston positive or negative?

(b) What is the difference $E_B - E_A$ of the internal energies of the system between the two macrostates A and B? Express your anwer in terms of the works W_{pis} and W_{stir}.

(c) Suppose that the pressure p_B of the gas in state B is *smaller* than its pressure p_A. Then the second step cannot be carried out since stirring the gas, while keeping its volume constant, will always result in increasing its pressure. How then could you proceed, with the same arrangement, to measure the energy difference $E_B - E_A$ between the two states A and B? <h-2> <a-11>

[C-4] *Heat absorbed in a combustion experiment*

A combustion experiment is performed on a system consisting of a mixture of fuel and oxygen. The system is enclosed by a container, of fixed volume, which is immersed in a water bath. As the fuel is burned, the temperature of the water changes and indicates that its internal energy has been increased.

(a) Has work been done on the system? If so, is this work positive negative?

(b) Has heat been absorbed by the system? If so, is it positive or negative?

(c) Has the internal energy of the system changed? If so, has it increased or decreased? <a-28>

[C-5] *Energy change resulting from gas expanding inside a container*

A container, with rigid wall and thermally insulated from its environment, is divided into two parts by a thin partion. As indicated in Fig. C-4a, the left half of the container is filled with a gas while the right half is empty. When a hole develops in the partition, the gas escapes so as to fill the entire container (as indicated in Fig. C-4b).

Is the final internal energy of the gas larger than, smaller than, or the same as its initial internal energy when it was in the left half of the container? Explain your reasoning. <h-11> <a-6>
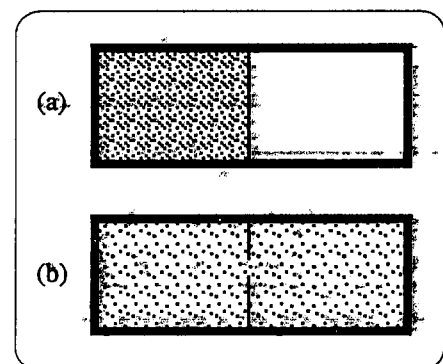

Fig. C-4. Gas escaping from the left half of a container into the entire container.

D. Applications of the thermodynamic energy law

The preceding section has indicated how all the quantities in the thermodynamic energy law

$$\Delta E = W + Q \qquad \text{(D-1)}$$

can be determined by macroscopic measurements. Hence we are now ready to examine some simple applications of this law.

Energy conservation of an isolated system. If a macroscopic system is at rest and isolated (i.e., if it interacts with no other systems), then no work is done on it and no heat is absorbed by it. Hence (D-1) implies that

$$\text{for an isolated system,} \quad \Delta E = 0. \qquad \text{(D-2)}$$

The internal energy of such an isolated system thus remains constant.

Energy changes of two interacting systems. Suppose that two systems X and X' interact with each other, but not with their environment. (For example, the systems might be the two gases illustrated in Fig. D-1.) The combined system X*, consisting of X and X', is then isolated so that its internal energy E* remains constant. But this energy is just equal to the sum of the internal energies E and E' of the two systems. Hence the conservation of energy of the combined system implies that

$$E^* = E + E' = \text{constant} \qquad \text{(D-3)}$$

or equivalently that

$$\Delta E^* = \Delta E + \Delta E' = 0. \qquad \text{(D-4)}$$

Interaction without heat. Consider the special case of a process where two systems interact without absorbing any heat, but do work on each other. (For example, suppose that the piston in Fig. D-1 is thermally insulated, but moves because of differing pressure forces on its two sides.) Such a process is called *adiabatic* in accordance with the following definition:

Def. | *Adiabatic process:* A process during which no heat (positive or negative) is absorbed by a system. | (D-5)

For example, a process can be made adiabatic if the system of interest is surrounded by thermally insulating material that prevents the system from absorbing heat.

Suppose that neither of the two interacting systems absorbs any heat. According to (D-1), the energy of each system changes then only because of the work done on it. Hence (D-4) implies that

$$W + W' = 0$$

or

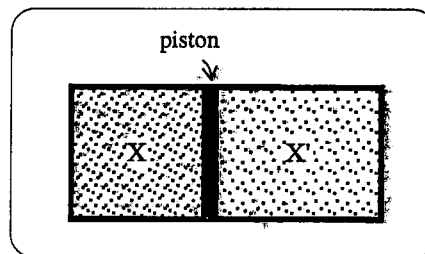$$W = -W' \qquad \text{(D-6)}$$

Fig. D-1. Two interacting gases X and X' separated by a movable piston. (The combined system X* is isolated.)

The relation (D-3) is true in the usual situation where the potential energy of interaction between the relatively few atoms near the touching surfaces of the systems is negligible compared to the much larger total energy of all the other atoms in the systems.

The word *adiabatic* comes from the Greek and means that heat is "not going through".

where W is the work done on system X by the other system and W' is the work done on system X' by the other system. Thus (D-6) merely asserts that the work W done on system X is equal to the work –W' done *by* system X'.

Interaction without work. Consider now another special case where no work is done by either system, but the systems can absorb heat from each other. (For example, suppose that the piston in Fig. D-1 is clamped in position, but that it allows heat to pass through.) According to (D-1), the energy of each system changes then only because of absorbed heat. Hence (D-4) implies that

$$Q + Q' = 0$$

or $$Q = -Q' \qquad\qquad (\text{D-7})$$

where Q is the heat absorbed by system X from the other system and Q' is the heat absorbed by system X' from the other system. Thus (D-7) merely asserts that the work Q absorbed by system X is equal to the heat –Q' *given off* by system X'.

General interaction. In general, a system can interact with other systems by many different processes. For example, there may be processes in which work is done, or heat is absorbed, or in which both occur. There may be processes in which the volume of a system remains constant, or its pressure remains constant, or its temperature remains constant. There may also be processes which occur rapidly or which occur so slowly as to be quasi-static.

However, suppose that a system changes from some macrostate A to some other macrostate B as a result of any such process. Then the thermodynamic energy law (D-1) implies that the change $E_B - E_A$ of the internal energy of the system is always the *same*, irrespective of the nature of the process leading from one state to the other.

Problems

[D-1] *Different processes compressing a gas*

A gas is thermally insulated from its surroundings. It is slowly compressed from an initial macrostate A, where its volume V_A is 4.00×10^{-3} m^3 and its pressure p_A is 1.00×10^5 N/m^2, to a final macrostate B where its volume V_B is 2.00×10^{-3} m^3. The final pressure p_B of the gas is then 3.17×10^5 N/m^2. The curved graph in Fig. D-2 indicates how the pressure p of the gas varies with its volume during this process. The work done on the gas during its compression is 352 J.

Alternatively, the gas can be brought to its final state B by successively carrying out the following two successive quasi-static processes. (1) The process A → C in which the pressure of the gas is kept constant at its initial value p_A while its volume is changed from V_A to V_B. (2) The process C → B in which the volume of the gas is kept constant at its value V_B while its pressure is changed from p_A to p_B.

(a) What is the work done on the gas during this entire process A → C → B?
(b) What is the heat absorbed by the gas during this entire process?

The following illustrates the solution of this problem.
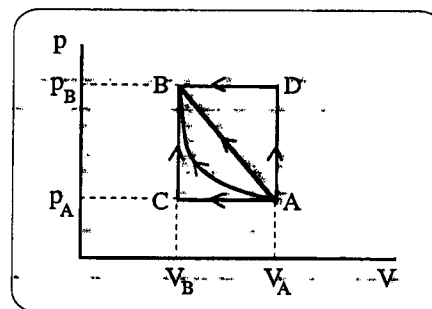

Fig. D-2. Different processes whereby a gas is changed from a state A to a state B.

Sample solution

Apply the energy law to the gas during the original process A → B when the gas was compressed while thermally insulated:

$$\Delta E = W + Q \tag{1}$$

or $\quad E_B - E_A = W + 0 = 352 \text{ J}. \tag{2}$

Work done on gas in process A → C:

$$W_{AC} = - p_A (V_C - V_A)$$

$$= - (1.00 \times 10^5 \text{ N/m}^2)(- 2.00 \times 10^{-3} \text{ m}^3) = 200 \text{ J}. \tag{3}$$

Work done on gas in process C → B:

$$W_{CB} = 0 \quad (\text{since } \Delta V = 0). \tag{4}$$

Work done in entire process A → C → B:

$$\boxed{W_{ACB} = W_{AC} + W_{CB} = 200 \text{ J}.} \quad \text{(Work done)} \tag{5}$$

Apply energy law to gas in process A → C → B:

$$E_B - E_A = W_{ACB} + Q_{ACB}$$

By (2) and (5):

$$352 \text{ J} = 200 \text{ J} + Q_{ACB}$$

or $\quad \boxed{Q_{ACB} = 152 \text{ J}.} \quad \text{(Heat absorbed)} \tag{6}$

[D-2] Different processes compressing a gas (continued)

Suppose that the gas of the preceding problem is brought to its initial state A to its final state B by successively carrying out the following two successive quasi-static processes illustrated Fig. D-2: (1) The process A → D in which the volume of the gas is kept constant at its initial value V_A while its pressure is changed from p_A to p_B. (2) The process D → B in which the pressure of the gas is kept constant at its value p_B while its volume is changed from V_A to V_B.

(a) What is the work done on the gas during this entire process A → D → B?
(b) What is the heat absorbed by the gas during this entire process? *<a-17>*

[D-3] Qualitative conclusion about absorbed heat

Suppose that the gas of Problem D-1 is quasi-statically brought from its initial state A to its final state B by a process during which the pressure of the gas varies linearly with its volume (as illustrated by the straight line AB in Fig. D-2).

(a) Is the heat absorbed by the gas in this process positive, negative, or zero? *<h-7> <a-23>*
(b) Explain the reasoning leading to your conclusion. *<a-2>*

[D-4] Vaporization of water

Water, placed in a cylinder closed off by a movable piston in contact with the surrounding atmosphere, is maintained at the constant atmospheric pressure of $1.013 \times 10^5 \text{ N/m}^2$. At a constant temperature of 100°C, 1.00 gram of water, occupying a volume of 1.00 cm^3, can then be transformed into 1.00 gram of water vapor occupying a volume of 1670 cm^3. To do this, 2260 J of heat must be added to the water.

(a) In this process, what is the work done on the water by the surrounding atmosphere?

(b) What is the difference $E_v - E_l$ between the internal energy E_v of one gram of water vapor and the internal energy E_l of one gram of liquid water? <a-14>

E. Summary

Definitions

Work: The work W done on a system is the macroscopoic work done on this system.

Heat: The heat Q absorbed by a system is the non-macroscopic work done on this system. (i.e., all the work that affects the energy of the system, but is not-macroscopically-apparent).

Thermometer: Any relatively small device characterized by a single macroscopic quantity (called its indicated "temperature") which changes when the device absorbs heat from some other system.

Quasi-static process: A process carried out so slowly that a system remains always very nearly in equilibrium

Adiabatic process: A process during which no heat (positive or negative) is absorbed by a system.

Important knowledge

Thermodynamic energy law (first law of thermodynamics):

$$\boxed{\Delta E = W + Q .}$$

(The change of internal energy ΔE between two macrostates of a system is thereby related to the work W done and the heat Q absorbed in any process connecting these states.)

Work done by pressure: Infinitesimal work $d'W$ done on a system, at pressure p, when its volume is quasi-statically changed by an infinitesimal amount dV:

$$d'W = - p\, dV .$$

New abilities

You should now be able to do the following:

(1) Find the work done on a fluid when its volume is slowly changed and its pressure is known at any instant.

(2) Apply the thermodynamic energy law to find information about the change of internal energy of a system, the work done on a system, or the heat absorbed by a system.

Problems

[E-1] *Heat and work while stirring a liquid*

A vessel contains a liquid and a paddle wheel. This wheel is connected by a string to a weight having a mass of 8.00 kg. When this weight descends with constant speed a distance of 3.00 m, it rotates the paddle wheel and thus stirs the liquid. The system consisting of the vessel and liquid has a constant volume and is thermally insulated from its surroundings.

(a) What is the change of the internal energy of this system when the weight descends by the specified amount?

(b) Suppose that the system is not thermally insulated, but remains throughout at the same constant temperature as its surroundings. What then is the change of the internal energy of this system when the weight descends by the specified amount?

(c) In the situation described in part *a*, what is the work done on the system? What is the heat absorbed by the system?

(d) In the situation described in part *b*, what is the work done on the system? What is the heat absorbed by the system? <a-33>

[E-2] *Work done on a melting ice cube*

The mass of 1.00 cm^3 of water is 1.00 gram, while the mass of 1.00 cm^3 of ice is 0.92 gram. Suppose that an ice cube, 1.00 inch on a side, melts. (1 inch = 2.54 cm)

(a) Does its volume then increase or decrease? Is the work done on the ice cube by the surrounding atmosphere then positive or negative?

(b) What is this work if the atmospheric pressure is $1.013 \times 10^5 \text{ N/m}^2$? <a-25>

[E-3] *Calorie and human energy consumption*

Heat was initially measured in terms of a unit called *calorie*. In the middle of last century it was realized that heat is a form of energy. Today the unit calorie is, therefore, defined so that

$$1 \text{ calorie} = 4.184 \text{ joule (exactly)}.$$

Although the unit calorie is rather obsolete, it is still used in the United States (although not in Canada and most other countries) to indicate the energy contents of foods.

A person who is not engaged in much strenuous activity, consumes daily food providing about 2,500 "large calories" (i.e., 2,500 kilocalories).

(a) What is the rate of energy consumption of such a person? Express your answer in terms of the unit *watt* (i.e., joule/second).

(b) Is this rate appreciably larger than, appreciably smaller than, or about the same as the rate of electrical energy consumption of a 100-watt light bulb? <a-30>

[E-4] *Measuring the internal energy of water*

A vessel, thermally insulated from its surroundings, contains water, a thermometer, and a paddle wheel for stirring the water. When the vessel contains 200 grams of water, the column of mercury in the thermometer rises by 1.3 cm when 5100 J of work is done to rotate the paddle wheel. When the vessel contains 300 grams of water, the column of mercury in the thermometer rises by the same amount when 7100 J of work is done to rotate the paddle wheel.

How much does the internal energy of 1.0 gram of water increase when its temperature change is indicated by a 1.3 cm rise in the mercury column of the thermometer? <h-4> <a-29>

[E-5] *Work done in a cyclic process*

As the volume of a fluid is slowly changed, its pressure p varies with its volume V in the manner indicated by the graph in Fig. E-1. The fluid thus undergoes the cyclic process ABCDA in which it starts in the indicated macrostate A and ends again in the same macrostate A.

Show that the magnitude of the work W, done by the pressure forces on the fluid in this process, is equal to the area within the closed curve ABCDA in the pressure-volume graph of Fig. E-1. *<a-20>*

[E-6] *Heat absorbed by a system in various processes*

Fig. E-2 shows how the pressure p of a system varies with its volume V as the system undergoes several quasi-static processes. When the system is brought from the state A to the state B along the indicated path ACB, the system absorbs 200 J of heat from its surroundings and does 75 J of work on its surroundings.

(a) When the system is brought from the state A to the state B along the path ADB, the system does 25 J of work on its surroundings. How much heat does the system absorb in this process?

(b) When the system is returned from B to A along the indicated curved path BA, 50 J of work is done on the system by its surroundings. How much heat does the system absorb in this process?

(c) It is known that the internal energy of the system in state D is larger than that in state A by 100 J. Use this information to find the heat absorbed by the system in the process A → D and in the process D → B. *<a-13>*

[E-7] *Work and heat in a cyclic process*

Fig. E-3 shows how the pressure p of a system varies with its volume V as the system is quasi-statically taken through the cyclic process ABCA.

(a) The internal energy of the system increases in the process A → B. Is the work done on the system in this process positive, negative, or zero? Is the heat absorbed by the system in this process positive, negative, or zero?

(b) The system absorbs some positive amount of heat in the process B → C. Is the work done on the system in this process positive, negative, or zero? Does the internal energy of the system in this process increase, decrease, or remain the same?

(c) In the process C → A, is the work done on the system positive, negative, or zero? Is the heat absorbed by the system in this process positive, negative, or zero? Does the internal energy of the system in this process increase, decrease, or remain the same?

(d) Is the heat absorbed by the system during the entire cyclic process ABCA positive, negative, or zero?

(e) The volume V_A of the system is 1.00×10^{-3} m^3 and its volume V_B is 3.00×10^{-3} m^3. The pressure p_A of the system is 2.00×10^5 N/m^2 and its pressure p_C is 4.00×10^5 N/m^2. What then is the numerical value of the heat absorbed by the system during the entire cyclic process ABCA? *<a-24>*

[E-8] *Internal energy change produced by an impact*

A lead block, having a mass of 1.20 kg, falls from a height of 30 m and hits a concrete floor. What is the resulting increase of the internal energy of the block if the heat transferred from the block to its surroundings is negligible? *<a-26>*

[E-9] *Heat and enthalpy change*

Chemical reactions cause a system to absorb an amount of heat Q from the surrounding atmosphere whose pressure has the constant value p. As a result of the reactions, the volume of the system changes by an amount ΔV.
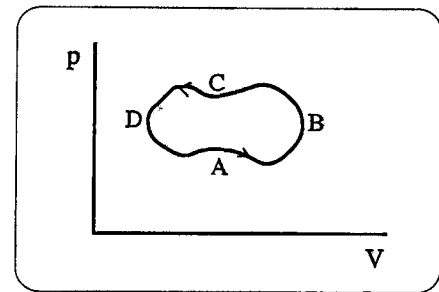

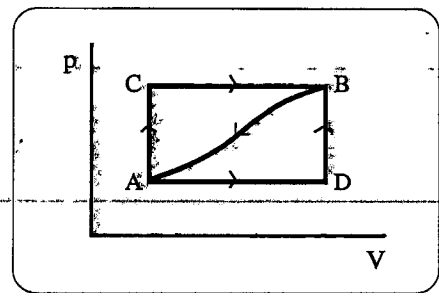Fig. E-1. A cyclic process represented on a graph of pressure versus volume.

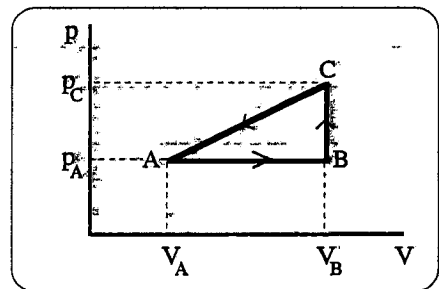Fig. E-2. Variation of pressure with volume in various processes of a system.

Fig. E-3. Variation of pressure with volume in a cyclic process.

(a) What is the change of the internal energy of the system as a result of these chemical reactions? Express your answer in terms of Q, p, and ΔV.

(b) The *enthalpy* H of the system is defined as the quantity $H = E + pV$ where E is the internal energy of the system and V is its volume. How is the heat Q absorbed by the system related to the change ΔH of its enthalpy? *<a-18>*

[E-10]† *Work done on a gas at constant temperature*

A gas in a cylinder is allowed to expand slowly while its temperature is maintained constant and heat is transferred to its surroundings. During this process the pressure p of the gas varies with its volume V in such a way that $p = C/V$ where C is some constant.

(a) Suppose that the gas starts in a state A, where its volume is V_A and its pressure is p_A, and then is allowed to expand until it reaches a volume V_B. What then is the pressure of the gas in this final state B?

(b) What is the work done on the gas during this expansion process?

The following illustrates the solution of this problem.

Sample solution

Since $pV = $ constant,

$$p_B V_B = p_A V_A$$

or $$\boxed{p_B = \frac{p_A V_A}{p_B}}.$$ (1)

Work on gas is the sum of all successive small works done in the process:

$$W = \int_{V_A}^{V_B} (-p\,dV) = \int_{V_A}^{V_B} \left(-\frac{C}{V}\,dV\right) = -C \int_{V_A}^{V_B} \frac{dV}{V}$$ (2)

where the symbol $\int_{V_A}^{V_B}$ denotes the sum (or *integral*) extending from the initial volume V_A to the final volume V_B. But each term in the sum can be expressed as an infinitesimal change, i.e., we can use our knowledge of logarithms (Appendix §) to write

$$\frac{dV}{V} = d(\ln V).$$

Hence $$W = -C \int_{V_A}^{V_B} d(\ln V) = -C\,[\ln V_B - \ln V_A]$$ (3)

since the sum of all successive small changes of the quantity $\ln V$ is just equal to the total change of this quantity. Thus

$$W = C\,[\ln V_A - \ln V_B] = C\,\ln\!\left(\frac{V_A}{V_B}\right).$$ (4)

But the constant $C = p_A V_A$. Hence

$$\boxed{W = p_A V_A \ln\!\left(\frac{V_A}{V_B}\right)}.$$ (5)

[E-11] *Work done on a gas at constant temperature (continued)*

Suppose that the gas in the preceding problem is slowly expanded, at constant temperature, from an initial volume $V_A = 1.00 \times 10^{-3}$ m^3 and pressure $p_A = 32 \times 10^4$ N/m^2, to a final volume $V_B = 8.00 \times 10^{-3}$ m^3.

(a) What then is the numerical value of the final pressure p_B of the gas?

(b) What is the numerical value of the work done on the gas in this process? *<a-27>*

[E-12]† *Work done on a gas in an adiabatic process*

A gas in a cylinder is allowed to expand slowly under conditions where no heat is transferred to its surroundings. In this process this pressure p of the gas varies with its volume V in such a way that $p = C/V^\gamma$ where γ (the Greek letter *gamma*) denotes some constant.

(a) Suppose that the gas starts in a state A, where its volume is V_A and its pressure is p_A, and then is allowed to expand until it reaches a volume V_B. What then is the pressure p_B of the gas in this final state B?

(b) What is the work done on the gas during this expansion process? *<h-6>* *<a-21>*

(c) Show that this work is equal to $[p_B V_B - p_A V_A]/(\gamma - 1)$.

[E-13] *Work done on a gas in an adiabatic process (continued)*

Suppose that the gas in the preceding problem is argon, for which the constant $\gamma = 5/3$. As in Problem E-8, this gas is slowly expanded from an initial volume $V_A = 1.00 \times 10^{-3}$ m^3 and pressure $p_A = 32 \times 10^4$ N/m^2, to a final volume $V_B = 8.00 \times 10^{-3}$ m^3. However, now this expansion process is carried out slowly under conditions where no heat is transferred to the gas.

(a) What then is the numerical value of the final pressure p_B of the gas?

(b) What is the numerical value of the work done on the gas in this process? *<a-16>*